C5.2 Elasticity and Plasticity

Lecture 0 — Introduction

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Hooke's Law

Robert Hooke (1678) wrote

> *"...it is...evident that the rule or law of nature in every springing body is that the force or power thereof to restore itself to its natural position is always proportionate to the distance or space it is removed therefrom, whether it be by rarefaction, or separation of its parts the one from the other, or by condensation, or crowding of those parts nearer together."*

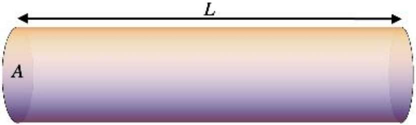
Hooke devised his law while designing clock springs, but noted that it appears to apply to all

> *"springy bodies whatsoever, whether metal, wood, stones, baked earths, hair, horns, silk, bones, sinews, glass and the like."*

Hooke's Law

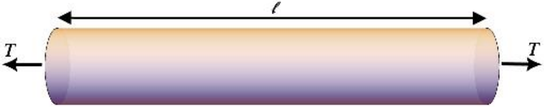
In a classical experiment, a solid rod is stretched to a tension T:

Before:



$A =$ cross-section area
$L =$ length

After:



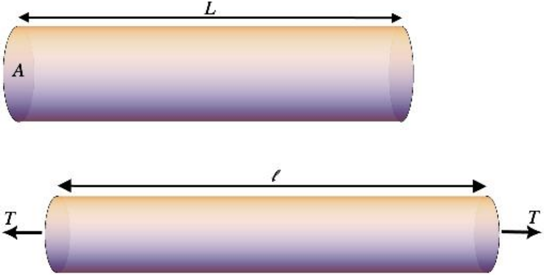
$T =$tension
$\ell =$new length

Hooke's Law states that tension $\propto$ extension:

$$T = k(\ell - L)$$

where constant of proportionality k measures stiffness of the rod.

Hooke's Law



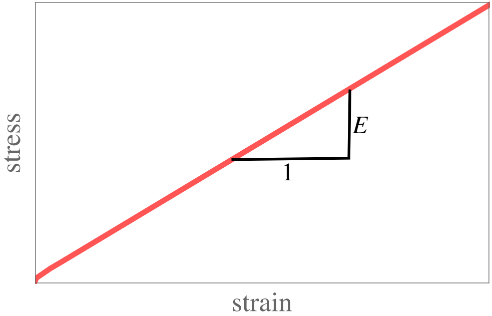
$$T = k(\ell - L)$$

Empirical observation that stiffness $k \propto A/L$ so

$$\left(\frac{T}{A}\right) = E\left(\frac{\ell - L}{L}\right) \qquad \text{i.e.} \qquad \text{stress} \propto \text{strain}$$

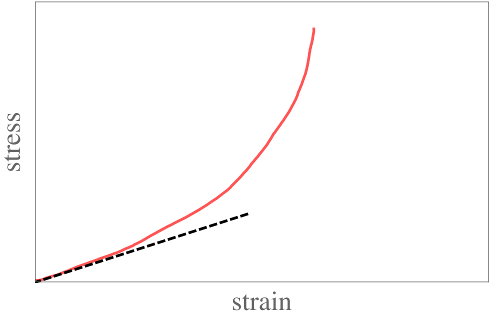
- E = *Young's modulus* is constant for any solid material
- T/A = *stress* i.e. force per unit area
- $(\ell - L)/L$ = *strain* i.e. extension relative to the initial length.

Hooke's law



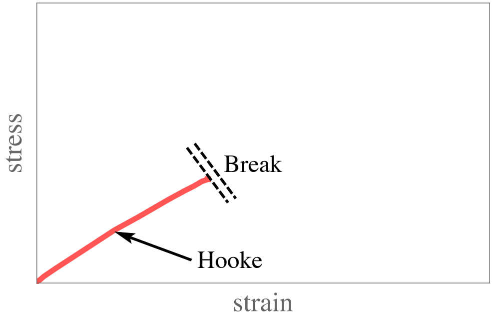
- Hooke's law underpins linear elasticity.
- It generally works well for sufficiently small strain but fails for larger strains.

Nonlinear elasticity



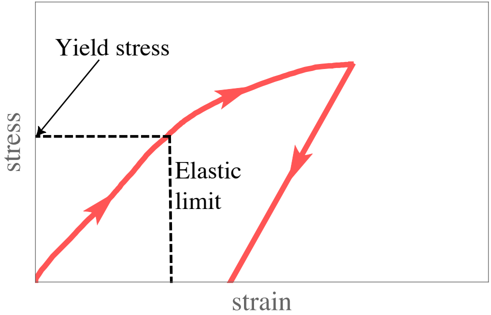
- Materials like rubber can undergo large strains with nonlinear stress–strain relationship.
- See *C5.1 Solid Mechanics*.
- But many (most?) solids are not like rubber — elasticity breaks down before nonlinearity becomes important.

Fracture



- Brittle solids fracture and break under excessive loading.
- Typically fracture occurs while material is still in the linear régime.

Plasticity



- Ductile solids undergo irreversible plastic deformation if applied stress exceeds a critical yield stress.
- When loading is removed, a permanent residual strain remains.

Lecture plan

Lectures	Lecture notes	Week
1–2	1 Equations of linear elasticity	1
3	2 Elementary steady solutions	2–3
4	3 Antiplane strain and torsion	
5	4 Plane strain	
6	5 Elastic waves	4–5
7–8	6 Models for thin structures	
9	7 Contact	6
10	8 Fracture	
11–13	9 Plasticity	7–8

Prerequisites

1. Basic techniques to solve ODEs and PDEs
   - *A1 Differential Equations 1*
   - *A6 Differential Equations 2*
   - *Prelims Fourier Series and PDEs*
   - [*B5.2 Applied Partial Differential Equations*]
2. Basic ideas from the calculus of variations
   - *ASO: Calculus of Variations*
3. Techniques from complex analysis
   - *A2: Metric Spaces and Complex Analysis*
   - *C5.6 Applied Complex Variables*
4. [Basic concept of the stress tensor]
   - *B5.3 Viscous Flow*
   - *C5.1 Solid Mechanics*