BO1 History of Mathematics
Lecture I
Introduction
Part 3: Napier's invention of logarithms

MT 2021 Week 1

A case study of a text from 1614

Napier's invention of logarithms:

- what did 17th-century mathematics look like?
- how can we begin to read historical texts?

Napier's definition of a logarithm (of a sine)

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Content:	what is it about? how is it written?
Significance:	why did it matter?
Historical Significance:	what new insight does this text offer us?

Context — who?

John Napier (1550–1617), Merchiston, Scotland

Scottish landowner with interests in:

- mining
- calculating aids
- astrology/astronomy
- The Revelation of St John

See *Oxford Dictionary of National Biography*: http://www.oxforddnb.com/view/article/19758

Context — why?

From Napier's preface to the translation of 1616:

> *Seeing there is nothing (right well-beloved Students of the Mathematics) that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances... I thought good heretofore to set forth in Latine for the publique use of Mathematicians.*

Context — why?

Inspired by the 16th-century technique of prosthaphaeresis:

the use of trigonometric identities such as

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$
$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

to convert multiplication into addition.

Context — in what form, and in which language?

Original Latin text of 1614:

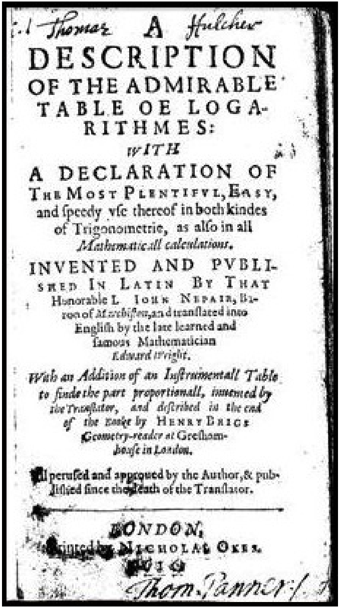
Mirifici logarithmorum canonis descriptio

translated into English by Edward Wright in 1616 as

A description of the admirable table of logarithms

Scanned text available via SOLO

Napier's 1616 title-page decoded


Thomas A Fulcher
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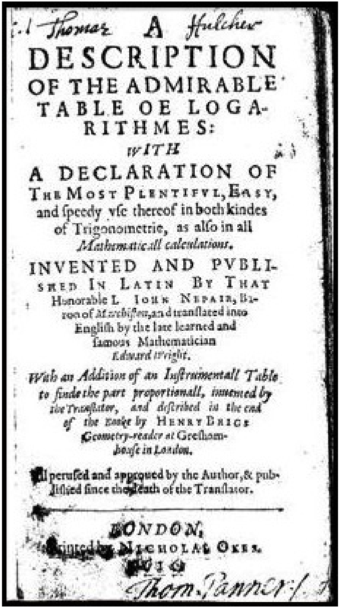

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Inventor:
- John Napier (1550–1617)

Translator:
- Edward Wright (?1558–1615) (interests: navigation, charts and tables)

Additional material:
- Henry Briggs (1561–1630) Gresham Professor of Geometry, later Savilian Professor of Geometry at Oxford (interests: navigation)

Printer:
- Nicholas Okes

Readers:
- Thomas Hulcher,
- Thomas Panner

Napier's logarithms: content

Recall:

> *The* Logarithme *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

Napier's logarithms

peare by the 19 *Prop.* 5. and 11. *Prop.* 7, *Euclid.*

3 *Def.* *Surd quantities, or vnexplicable by number, are said to be defined, or expressed by numbers very neere, when they are defined or expressed by great numbers which differ not so much as one vnite from the true value of the Surd quantities.*

As for example. Let the semidiameter, or whole sine be the rational number 10000000 the sine of 45 degrees shall be the square root of 50, 000, 000,000,000, which is surd, or irrationall and inexplicable by any number, & is included between the limits of 7071067 the lesse, and 7071068 the greater: therfore, it differeth not an vnite from either of these. Therefore that surd sine of 45 degrees, is said to be defined and expressed very neere, when it is expressed by the whole numbers, 7071067, or 7071068, not regarding the fractions. For in great numbers there ariseth no sensible error, by neglecting the fragments, or parts of an vnite.

4 *Def.* *Equall-timed motions are those which are made together, and in the same time.*

As in the figures following, admit that B be moued from A to C, in the same time, wherin *b* is moued from *a* to *c* the right lines A C & *a c*, shall be sayd to be described with an equall-timed motion.

5 *Def.* *Seeing that there may bee a slower and a swifter motion giuen then any motion, it shall necessarily follow, that there may be a motion giuen of equall swiftnesse to any motion (which wee define to be neither swifter nor slower.)*

6 *Def.* The Logarithme *therfore of any sine is a number very neerely expressing the line, which increased*

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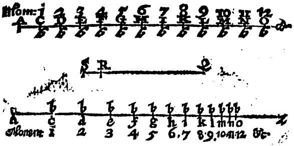
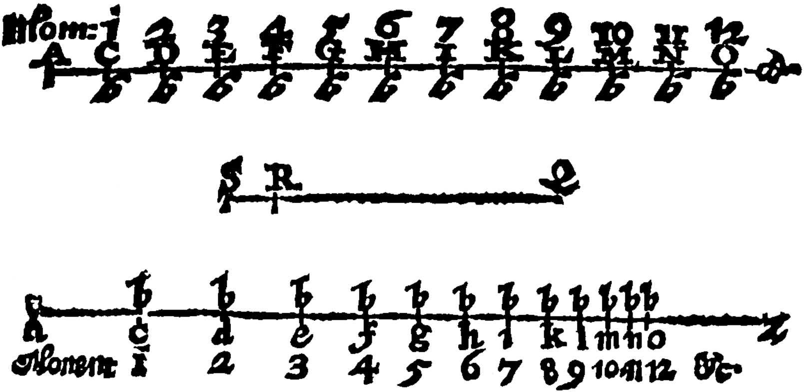


As for example. Let the 2 figures going afore bee here repeated, and let B bee moued alwayes, and euery where with equall, or the same swiftnesse wherewith *b* beganne to bee moued in the beginning, when it was in *a*. Then in the first moment let B proceed from A to C, and in the same time let *b* moue proportionally from *a* to *c*, the number defining or expressing A C shal be the *Logarithme* of the line, or sine *c* Z. Then in the second moment let B bee moued forward from C to D. And in the same moment or time let *b* be moued proportionally from *c* to *d*, the number definining A D, shall bee the *Logarithme* of the sine *d* Z. So in the third moment let B go forward equally from D to E, and in the same moment let *b* be moued forward proportionally from *d* to *e*, the number expressing A E the *Logarithme* of the sine *e* Z. Also in the fourth moment, let B proceed

Napier's logarithms



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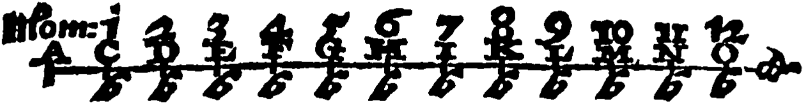

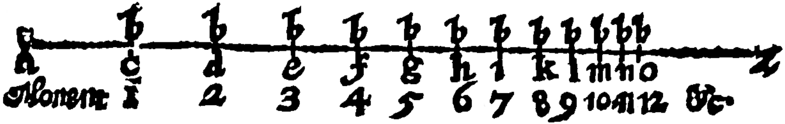

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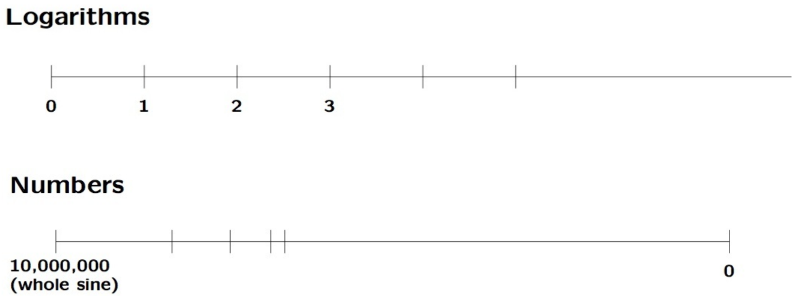

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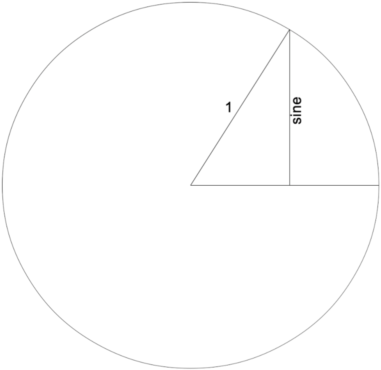

Sine of angle at centre varies between 0 and ± 1 as the labelled radius sweeps around the circle

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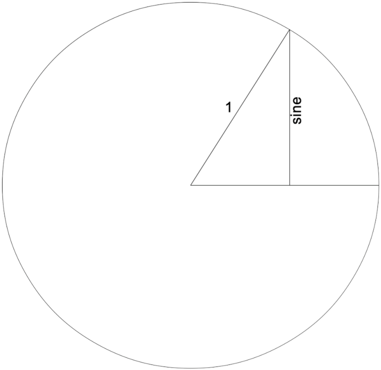

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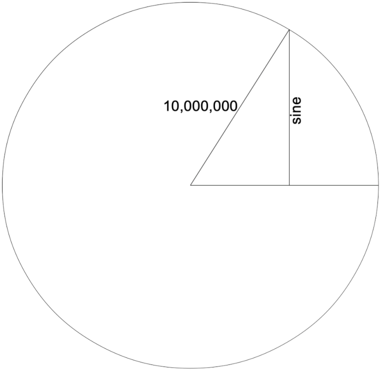
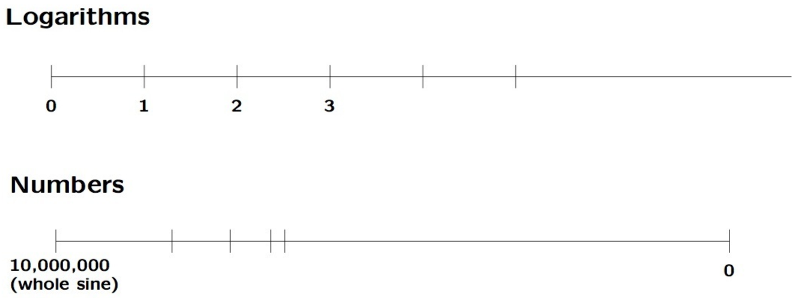


Sine of angle at centre varies between 0 and $\pm 10,000,000$ as the labelled radius sweeps around the circle

Napier's logarithms



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In modern terms (i.e., not Napier's):

$$\text{if } y = 10^7\left(1 - 10^{-7}\right)^x, \text{ then } \operatorname{Nap}\log y = x$$

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No notion of base, although Nap log 'nearly' has base $\frac{1}{e}$ — see: Robin Wilson, *Euler's Pioneering Equation*, OUP, 2019, p. 101

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Definition revised to remove the need to subtract $\operatorname{Nap}\log 1$

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Briggs produced *Logarithmorum chilias prima* (*The first thousand logarithms*) in 1617, followed by his *Arithmetica logarithmica* in 1624, which contained logarithms of 1 to 20,000 and 90,000 to 100,000, all to 14 decimal places (calculated by hand); the gap in the table was filled by Adriaan Vlacq in 1628

Napier's logarithms

One last time:

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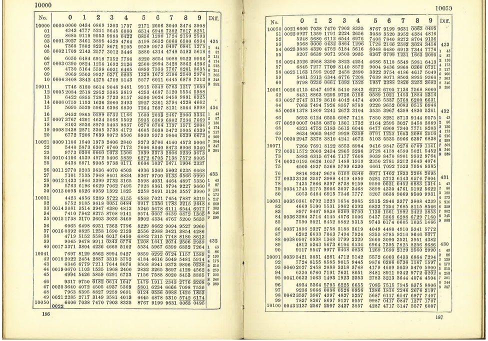
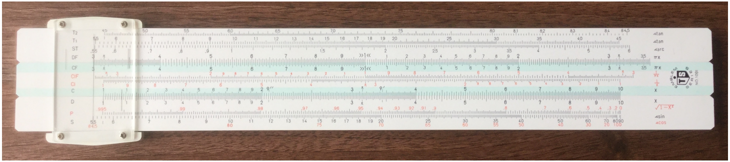
Significance



Napier's logarithms:

- caught on very quickly
- a calculating aid (until the 1980s)
- logarithms rapidly came to have other interpretations (as you know, and as we shall see)



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- Importance of informal/social communication, alongside published texts