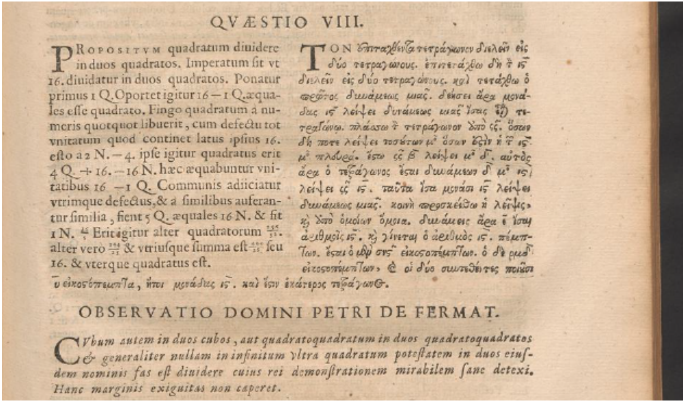
# B01.1 History of Mathematics
## Course Context Session



QVÆSTIO VIII.

PROPOSITVM quadratum diuidere in duos quadratos. Imperatum sit vt 16. diuidatur in duos quadratos. Ponatur primus 1 Q. Oportet igitur 16 − 1 Q. æquales esse quadrato. Fingo quadratum à numeris quotquot libuerit, cum defectu tot vnitatum quod continet latus ipsius 16. esto à 2 N. − 4. ipse igitur quadratus erit 4 Q. + 16. − 16 N. hæc æquabuntur vnitatibus 16 − 1 Q. Communis adiiciatur vtrimque defectus, & à similibus auferantur similia, fient 5 Q. æquales 16 N. & fit 1 N. $\frac{16}{5}$ Erit igitur alter quadratorum $\frac{256}{25}$. alter verò $\frac{144}{25}$ & vtriusque summa est $\frac{400}{25}$ seu 16. & vterque quadratus est.

ΤΟΝ ἐπιταχθέντα τετράγωνον διελεῖν εἰς δύο τετραγώνους. ἐπιτετάχθω δὴ τὸν ις̄ διελεῖν εἰς δύο τετραγώνους. καὶ τετάχθω ὁ πρῶτος δυνάμεως μιᾶς. δεήσει ἄρα μονάδας ις̄ λείψει δυνάμεως μιᾶς ἴσας εἶναι τετραγώνῳ. πλάσσω τὸν τετράγωνον ἀπὸ ςς̄. ὅσων δή ποτε λείψει τοσούτων μο ὅσων ἐστὶν ἡ τῶν ις̄ μο πλευρά. ἔστω ςς̄ β̄ λείψει μο δ̄. αὐτὸς ἄρα ὁ τετράγωνος ἔσται δυνάμεων δ̄ μο ις̄ λείψει ςς̄ ις̄. ταῦτα ἴσα μονάσι ις̄ λείψει δυνάμεως μιᾶς. κοινὴ προσκείσθω ἡ λεῖψις, καὶ ἀπὸ ὁμοίων ὅμοια. δυνάμεις ἄρα ε̄ ἴσαι ἀριθμοῖς ις̄. καὶ γίνεται ὁ ἀριθμὸς ις̄ πέμπτων. ἔσται ὁ μὲν σνς̄ εἰκοστοπέμπτων. ὁ δὲ ρμδ̄ εἰκοστοπέμπτων, καὶ οἱ δύο συντεθέντες ποιοῦσιν ῡ εἰκοστόπεμπτα, ἤτοι μονάδας ις̄. καὶ ἔστιν ἑκάτερος τετράγωνος.

OBSERVATIO DOMINI PETRI DE FERMAT.

*Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum vltra quadratum potestatem in duos eiusdem nominis fas est diuidere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.*

Dr. Brigitte Stenhouse

15 October 2021

# When do we meet mathematicians?

Cauchy's Theorem

Kovaleskaya's Theorem

Liouville's Theorem

Weierstrass M-Test

Fermat's Last Theorem

Noether's Theorem

Lagrange's Theorem

Fermat's Little Theorem

Stokes-Cartan Theorem

Rolle's Theorem

Gauss's Theorem

Green's Theorem

Taylor Series

Ostrogradsky's Theorem

Hardy–Ramanujan Theorem

# Outline of talk

- Reading historical mathematics
- A theorem in disguise
- Who gets a theorem named after them?
- Benefits of studying history of maths (for maths students)

# Guess the theorem...



## SECTION IX.

*Solution de quelques Problêmes qui dépendent des Méthodes précédentes.*

### PROPOSITION I.

### Problême.

163. SOIT *une ligne courbe* AMD (AP = x, PM = y, AB = a) *telle que la valeur de l'appliquée* y *ſoit exprimée par une fraction, dont le numérateur & le dénominateur deviennent chacun zero lorſque* x = a, *c'eſt-à-dire lorſque le point* P *tombe ſur le point donné* B. *On demande quelle doit être alors la valeur de l'appliquée* BD. FIG. 130.

Soient entenduës deux lignes courbes *ANB*, *COB*, qui ayent pour axe commun la ligne *AB*, & qui ſoient telles que l'appliquée *PN* exprime le numérateur, & l'appliquée *PO* le dénominateur de la fraction générale qui convient à toutes les *PM* : de ſorte que $PM = \frac{AB \times PN}{PO}$. Il eſt clair que ces deux courbes ſe rencontreront au point *B* ;

clair que ces deux courbes ſe rencontreront au point *B* ;
puiſque par la ſuppoſition *PN* & *PO* deviennent chacune zero lorſque le point *P* tombe en *B*. Cela poſé, ſi l'on imagine une appliquée *bd* infiniment proche de *BD*, & qui rencontre les lignes courbes *ANB*, *COB* aux points *f*, *g* ; l'on aura $bd = \frac{AB \times bf}{bg}$, laquelle * ne différe pas de *BD*. **Art. 2.* Il n'eſt donc queſtion que de trouver le rapport de *bg* à *bf*. Or il eſt viſible que la coupée *AP* devenant *AB*, les appliquées *PN*, *PO* deviennent nulles ; & que *AP* devenant *Ab*, elles deviennent *bf*, *bg*. D'où il ſuit que ces appliquées, elles-mêmes *bf*, *bg*, ſont la différence des appliquées en *B* & *b* par rapport aux courbes *ANB*, *COB* ; & partant que ſi l'on prend la différence du numérateur, & qu'on la diviſe par la différence du dénominateur, aprés

T

# Guess the theorem...

Let $AMD$ be a curved line ($AP = x$, $PM = y$, $AB = a$) such that the value of the ordinate $y$ is expressed by a fraction, whose numerator and denominator each become zero when $x = a$, that is to say when the point $P$ coincides with the given point $B$. We ask what should be the value of the ordinate BD.

Let there be two curved lines $ANB$, $COB$, which have the axis $AB$ in common, & which are such that the ordinate $PN$ expresses the numerator, & the ordinate $PO$ the denominator of the general fraction which agrees with $PM$: such that $PM = \frac{AB \times PN}{PO}$. It is clear that these two curves meet each other at the point $B$; as by the assumption $PN$ & $PO$ each become zero when the the point $P$ coincides with $B$. That being said, if we imagine an ordinate $bd$ infinitely close to $BD$, & which meets the curved lines $ANB$, $COB$, at the points $f$, $g$; we will have $bd = \frac{AB \times bf}{bg}$, which does not differ from $BD$.

It only remains to find the relationship between $bg$ and $bf$. It is clear that the line $AP$ becomes $AB$, the ordinates $PN$, $PO$ become zero; & that $AP$ becomes $Ab$. From where it follows that these ordinates, themselves $bf$, $bg$, are the difference of the ordinates in $B$ & $b$ with respect to the curves $ANB$, $COB$; & thus if we take the difference of the numerator, & divide it by the difference of the denominator, after making $x = a = Ab$ or $AB$, we will have the desired value of the ordinate $bd$ or $BD$.

Example 1

164. Let $y = \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{aax}}{a - \sqrt[4]{ax^3}}$. It is clear that when $x = a$, the numerator and the denominator of the fraction will each become equal to zero. It is why we take the difference $\frac{a^3dx - 2x^3dx}{\sqrt{2a^3x - x^4}} - \frac{aadx}{3\sqrt[3]{axx}}$ of the numerator, & we divide it by the difference $-\frac{3adx}{4\sqrt[4]{a^3x}}$ of the denominator, setting $x = a$, that is to say we divide $-\frac{4}{3}adx$ by $-\frac{3}{4}dx$; this gives $\frac{16}{9}a$ for the sought after value of BD.

(*Analyse des Infiniment Petits...*, 1696. Page 145–6. Translation my own).

### Exemple I.

164. Soit $y = \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{aax}}{a - \sqrt[4]{ax^3}}$. Il eſt clair que lorſque $x = a$, le numérateur & le dénominateur de la fraction deviennent égaux chacun à zero. C'eſt-pourquoy l'on prendra la différence $\frac{a^3dx - 2x^3dx}{\sqrt{2a^3x - x^4}} - \frac{aadx}{3\sqrt[3]{axx}}$ du numérateur, & on la diviſera par la différence $-\frac{3adx}{4\sqrt[4]{a^3x}}$ du dénominateur, aprés avoir fait $x = a$, c'eſt-à-dire qu'on diviſera $-\frac{4}{3}adx$ par $-\frac{3}{4}dx$; ce qui donne $\frac{16}{9}a$ pour la valeur cherchée de $BD$.

# L'Hôpital's Rule

What was missing?

**Theorem (L'Hôpital's Rule)**

*Suppose $f$, $g$ are continuous on $(a-\delta,\, a+\delta)$ (for some $\delta > 0$) and differentiable on $(a-\delta, a+\delta) \setminus \{a\}$, and $f(a) = g(a) = 0$ then*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

*provided that the limit on the right-hand side exists.*

### Theorem (L'Hôpital's Rule)

*Suppose $f$, $g$ are continuous on $(a-\delta,\ a+\delta)$ (for some $\delta > 0$) and differentiable on $(a-\delta, a+\delta) \setminus \{a\}$, and $f(a) = g(a) = 0$ then*

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

*provided that the limit on the right-hand side exists.*

- L'Hôpital was looking at curves, not functions

### Theorem (L'Hôpital's Rule)

*Suppose $f$, $g$ are continuous on $(a-\delta,\ a+\delta)$ (for some $\delta > 0$) and differentiable on $(a-\delta, a+\delta) \setminus \{a\}$, and $f(a) = g(a) = 0$ then*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

*provided that the limit on the right-hand side exists.*

- L'Hôpital was looking at curves, not functions
- Not all functions are differentiable. What conditions do we need?

### Theorem (L'Hôpital's Rule)

*Suppose $f$, $g$ are continuous on $(a-\delta, a+\delta)$ (for some $\delta > 0$) and differentiable on $(a-\delta, a+\delta) \setminus \{a\}$, and $f(a) = g(a) = 0$ then*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

*provided that the limit on the right-hand side exists.*

- L'Hôpital was looking at curves, not functions
- Not all functions are differentiable. What conditions do we need?
- What is a limit? Are there different types of convergence? What conditions do we need?

### Theorem (L'Hôpital's Rule)

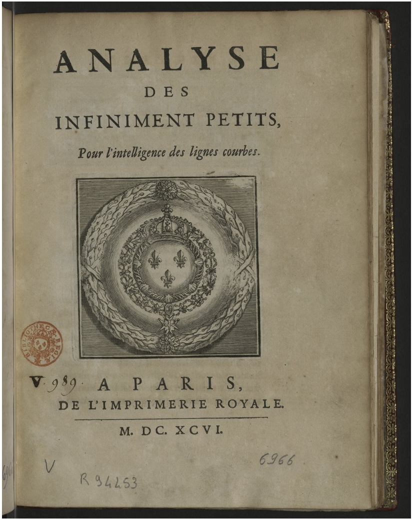
*Suppose $f$, $g$ are continuous on $(a-\delta,\ a+\delta)$ (for some $\delta > 0$) and differentiable on $(a-\delta, a+\delta) \setminus \{a\}$, and $f(a) = g(a) = 0$ then*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

*provided that the limit on the right-hand side exists.*

- L'Hôpital was looking at curves, not functions
- Not all functions are differentiable. What conditions do we need?
- What is a limit? Are there different types of convergence? What conditions do we need?
- Where did this notation come from?

# A theorem belonging to L'Hôpital?

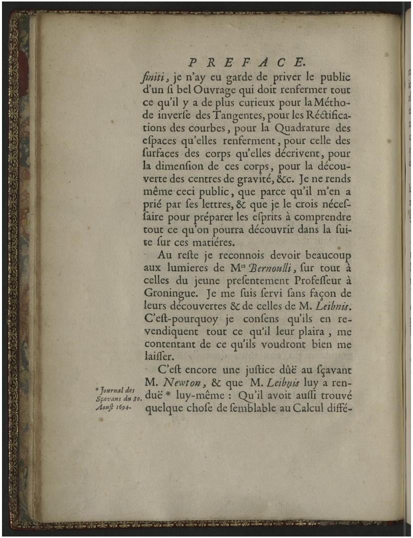


ANALYSE
DES
INFINIMENT PETITS,
*Pour l'intelligence des lignes courbes.*


A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DC. XCVI.



Source gallica.bnf.fr / Bibliothèque nationale de France



PREFACE.

*finiti*, je n'ay eu garde de priver le public d'un ſi bel Ouvrage qui doit renfermer tout ce qu'il y a de plus curieux pour la Méthode inverſe des Tangentes, pour les Réctifications des courbes, pour la Quadrature des eſpaces qu'elles renferment, pour celle des ſurfaces des corps qu'elles décrivent, pour la dimenſion de ces corps, pour la découverte des centres de gravité, &c. Je ne rends même ceci public, que parce qu'il m'en a prié par ſes lettres, & que je le crois néceſſaire pour préparer les eſprits à comprendre tout ce qu'on pourra découvrir dans la ſuite ſur ces matiéres.

Au reſte je reconnois devoir beaucoup aux lumieres de M[rs] *Bernoulli*, ſur tout à celles du jeune preſentement Profeſſeur à Groningue. Je me ſuis ſervi ſans façon de leurs découvertes & de celles de M. *Leibnis*. C'eſt-pourquoy je conſens qu'ils en revendiquent tout ce qu'il leur plaira, me contentant de ce qu'ils voudront bien me laiſſer.

C'eſt encore une juſtice dûë au ſçavant M. *Newton*, & que M. *Leibnis* luy a renduë * luy-même : Qu'il avoit auſſi trouvé quelque choſe de ſemblable au Calcul diffé-

* *Journal des Sçavans du 30. Aouſt 1694.*

Source gallica.bnf.fr / Bibliothèque nationale de France

# A theorem belonging to L'Hôpital?

Guillaume-François-Antoine Marquis de l'Hôpital, Marquis de Sainte-Mesme, Comte d'Entremont and Seigneur d'Ouques-la-Chaise (1661–1704)

Johann Bernoulli (1667–1748)

# A theorem belonging to L'Hôpital?

> *I will be happy to give you a retainer of 300 pounds, beginning with the first of January of this year. ... I promise shortly to increase this retainer, which I know is very modest, as soon as my affairs are somewhat straightened out. ... I am not so unreasonable as to demand in return all of your time, but I will ask you to give me at intervals some hours of your time to work on what I request and also to communicate to me your discoveries, at the same time asking you not to disclose any of them to others. I ask you even not to send here to Mr Varignon or to others any copies of the writings you have left with me; if they are published, I will not be at all pleased. Answer me regarding all this ...*

L'Hôpital to Bernoulli, 17 March 1694

# A theorem belonging to L'Hôpital?

> *I will be happy to give you a retainer of 300 pounds, beginning with the first of January of this year. ... I promise shortly to increase this retainer, which I know is very modest, as soon as my affairs are somewhat straightened out. ... I am not so unreasonable as to demand in return all of your time, but I will ask you to give me at intervals some hours of your time to work on what I request and also to communicate to me your discoveries, at the same time asking you not to disclose any of them to others. I ask you even not to send here to Mr Varignon or to others any copies of the writings you have left with me; if they are published, I will not be at all pleased. Answer me regarding all this ...*

L'Hôpital to Bernoulli, 17 March 1694

> *You have only to let me know your definite wishes, if I am to publish nothing more in my life, for I will follow them precisely and nothing more by me will be seen.*

Bernoulli to L'Hôpital, 1695

# Context, content, significance

# Context, content, significance

Context: who? when? where? why?

# Context, content, significance

Context: who? when? where? why?

Content: what is it about? how is it written?

# Context, content, significance

Context: who? when? where? why?

Content: what is it about? how is it written?

Significance: why did it matter?

# Context, content, significance

| | |
|---|---|
| Context: | who? when? where? why? |
| Content: | what is it about? how is it written? |
| Significance: | why did it matter? |
| Historical Significance: | what new insight does this text offer us? |

Behind this diversity you describe, is mathematics all one?

*That's the point. Take, for example, the so-called Pythagorean theorem. In the distant past, it could be seen as a statement that expressed a relationship between the lengths of the sides of a right-angled triangle, understood as geometric objects, and it could also be seen as a procedure that, from two numbers that expressed the lengths of two sides, allowed us to produce the third. Today, we move from one of these ways of seeing to the other in a very flexible way, so flexible that we no longer see the different statements that have been synthesized under the name of "Pythagorean theorem"... What did it take, what circulation of ideas, what transformations of ways of seeing the triangle, for these different statements to become points of view between which one could easily move? So my answer to the question: "Is mathematics all one?" is yes and no. Because in fact, what for us today is one and the same thing could have been understood in different ways in the past; and it could also appear tomorrow as the same thing as something that today seems to us to have no relation. It has been a long work in history for those things to be made, and the history of this work is just as interesting as the history of the introduction of a new concept or of the achievement of a new theorem.*

Interview with Prof. Karine Chemla:
https://www.insmi.cnrs.fr/en/cnrsinfo/8ecm-interview-karine-chemla

# In Summary...

Using History of Mathematics to broaden ideas of what constitutes valuable mathematical practice, can enable you to:

- Develop ‘Mathematical Resilience’.

# In Summary...

Using History of Mathematics to broaden ideas of what constitutes valuable mathematical practice, can enable you to:

- Develop 'Mathematical Resilience'.
- Practice thinking critically about mathematics and mathematical proofs.

# In Summary...

Using History of Mathematics to broaden ideas of what constitutes valuable mathematical practice, can enable you to:

- Develop 'Mathematical Resilience'.
- Practice thinking critically about mathematics and mathematical proofs.
- Naturally meet a wider diversity of mathematicians.

# In Summary...

Using History of Mathematics to broaden ideas of what constitutes valuable mathematical practice, can enable you to:

- Develop 'Mathematical Resilience'.
- Practice thinking critically about mathematics and mathematical proofs.
- Naturally meet a wider diversity of mathematicians.
- Study history, sociology, politics, linguistics, translation, philosophy. . .

# In Summary...

Using History of Mathematics to broaden ideas of what constitutes valuable mathematical practice, can enable you to:

- Develop ‘Mathematical Resilience’.
- Practice thinking critically about mathematics and mathematical proofs.
- Naturally meet a wider diversity of mathematicians.
- Study history, sociology, politics, linguistics, translation, philosophy. . .
- Understand how mathematics can be presented to different audiences.

# In Summary...

Using History of Mathematics to broaden ideas of what constitutes valuable mathematical practice, can enable you to:

- Develop ‘Mathematical Resilience’.
- Practice thinking critically about mathematics and mathematical proofs.
- Naturally meet a wider diversity of mathematicians.
- Study history, sociology, politics, linguistics, translation, philosophy. . .
- Understand how mathematics can be presented to different audiences.

# References and sources

Podcasts: You're Dead To Me: Medieval Science, from 24 September 2021, on BBC Sounds; In our time: e, from 25 September 2014, on BBC Sounds

Twitter: @MathHistFacts, @MathsHistory

**Bibliography**


G. F. A. M. de L'Hôpital. *Analyse des infiniment petits, pour l'intelligence des lignes courbes*. l'Imprimerie Royale Paris, 1696. Find on Solo here.

C. Truesdell. The new Bernoulli edition. *Isis*, 49:54–62, 1958. Find on Solo here.