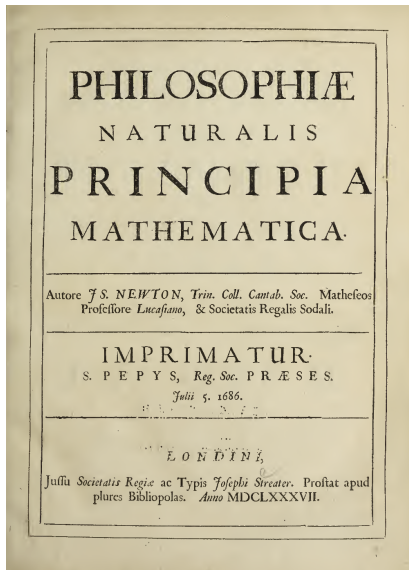


BO1 History of Mathematics  
Lecture IV  
Newton's *Principia*  
Part 3: The *Principia*

MT 2021 Week 2

Isaac Newton: *The mathematical principles of natural philosophy* (London, 1687)



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# The laws of motion

[ 12 ]

## AXIOMATA SIVE LEGES MOTUS

LEX. I.

*Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

**P**rojectilia perseverant in motibus suis nisi quatenus a resistentia aeris retardantur & vi gravitatis impelluntur deorsum.

Trochus, cujus partes coherendo perpetuo retrahunt sese a motibus rectilinis, non cessat rotari nisi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus suos & progressivos & circulares in spatii minus resistentibus factos conservant diutius.

LEX. II.

*Mutationem motus proportionalem esse vi motrici impressae, & fieri secundum lineam rectam qua vis illa imprimitur.*

Si vis aliqua motum quemvis generet, dupla duplum, tripla tripulum generabit, sive simul & semel, sive gradatim & successive impressa fuerit. Et hic motus quoniam in eandem semper plagam cum vi generatrice determinatur, si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel obliquo oblique adjicitur, & cum eo secundum utriusque determinationem componitur.

LEX. III.

[ 13 ]

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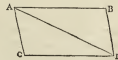
*Actiōi contrariam semper & aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales & in partes contrarias dirigi.*

Quicquid premit vel trahit alterum, tantundem ab eo premitur vel trahitur. Siquis lapidem digito premit, premitur & hujus digitus a lapide. Si equus lapidem funi allegatum trahit, retrahetur etiam & equus aequaliter in lapidem: nam funis utrinque distentus eodem relaxandi se conatu urgebit Equum versus lapidem, ac lapidem versus equum, tantumque impedit progressum unius quantum promovet progressum alterius. Si corpus aliquod in corpus aliud impingens, motum ejus vi sua quomodocumque mutaverit, idem quoque vicissim in motu proprio eandem mutationem in partem contrariam vi alterius (ob aequalitatem pressiois mutuae) subibit. His actionibus aequales sunt mutationes non velocitatum sed motuum, (scilicet in corporibus non aliunde impeditis: ) Mutationes enim velocitatum, in contrarias itidem partes factae, quia motus aequaliter mutantur, sunt corporibus reciproce proportionales.

COROL. I.

*Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore describere, quo latera separatis.*

Si corpus dato tempore, vi sola *M*, ferretur ab *A* ad *B*, & vi sola *N*, ab *A* ad *C*, compleatur parallelogrammum *ABDC*, & vi utraque ferretur id eodem tempore ab *A* ad *D*. Nam quoniam vis *N* agit secundum lineam



*AC* ipsi *BD* parallelam, hac vi nihil mutabit velocitatem accedendi ad lineam illam *BD* a vi altera genitam. Accedet igitur corpus eodem tempore ad lineam *BD* sive vi *N* imprimatur, sive non, atque adeo in fine illius temporis reperietur alicubi in linea



# Book I: Motion of bodies

Book I, Section I: On the method of first and last ratios

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**Lemma I:** Quantities, and ratios of quantities, which [...] approach nearer to each other than by any given difference, become ultimately equal.

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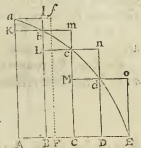
For suppose they are ultimately unequal, and their ultimate difference is  $D$ . Then they cannot approach nearer to equality than by that difference.

# Book I, Lemma II

[ 27 ]

## Lemma II.

Si in figura quavis  $Aa cE$  rectis  $Aa$ ,  $AE$ , & curva  $AcE$  comprehensa, inscribantur parallelogramma quotcumq;  $Ab$ ,  $Bc$ ,  $Cd$ , &c. sub basibus  $AB$ ,  $BC$ ,  $CD$ , &c. equalibus, & lateribus  $Bb$ ,  $Cc$ ,  $Dd$ , &c. figuræ lateri  $Aa$  parallelis contenta; & compleantur parallelogramma  $aKbl$ ,  $bLcm$ ,  $cMdn$ , &c. Dein horum parallelogrammorum latitudo minuitur, & numerus augeatur in infinitum: dico quod ultime rationes, quas habent ad se invicem figura inscripta  $AKbLcMdD$ , circumscripta  $Aalbmcndoe$ , & curvilinea  $AabedE$ , sunt rationes equalitatis.



Nam figuræ inscriptæ & circumscriptæ differentia est summa parallelogrammorum  $Kl + Lm + Mn + Do$ , hoc est (ob æquales omnium bases) rectangulum sub unius basi  $Kb$  & altitudinum summa  $Aa$ , id est rectangulum  $ABla$ . Sed hoc rectangulum, eo quod latitudo ejus  $AB$  in infinitum minuitur, fit minus quovis dato. Ergo, per Lemma I, figura inscripta & circumscripta & multo magis figura curvilinea intermedia fiunt ultimo æquales. *Q. E. D.*

## Lemma III.

Eadem rationes ultime sunt etiam equalitatis, ubi parallelogrammorum latitudines  $AB$ ,  $BC$ ,  $CD$ , &c. sunt inæquales, & omnes minuantur in infinitum.

Sit enim  $AF$  æqualis latitudini maxima, & compleatur parallelogrammum  $FAaf$ . Hoc erit majus quam differentia figuræ inscriptæ & si usque circumscriptæ, at latitudine sua  $AF$

**Lemma II:** Ultimate equality of inscribed figure, circumscribed figure, and curved area

# Motion under centripetal forces

[ 37 ]

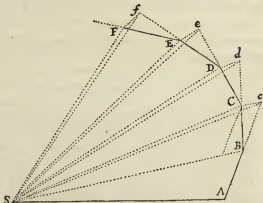
## SECT. II.

### De Inventione Virium Centripetarum.

#### Prop. I. Theorema. I.

*Areas quas corpora in gyros acta radiis ad immobile centrum virium ductis describunt, & in planis immobilibus consistere, & esse temporibus proportionales.*

Dividatur tempus in partes æquales, & prima temporis parte describat corpus vi insita rectam  $AB$ . Idem secunda temporis parte, si nil impediret, recta pergeret ad  $e$ , (per Leg. I) describens lineam  $Be$  æqualem ipsi  $AB$ , adeo ut radii  $AS$ ,  $BS$ ,  $eS$  ad centrum actis, confectæ forent æquales arcæ  $SB$ ,  $BSe$ . Verum ubi corpus venit ad  $B$ , agat vis centripeta impulsu unico sed magno, faciatq; corpus a recta  $Be$  deflectere & pergere in recta  $BC$ . Ipsi  $BS$  parallela agatur  $eC$  occurrens  $BC$  in  $C$ , & completa secunda temporis parte, corpus (per Legum Corol. 1) reperietur in  $C$ , in eodem plano cum triangulo  $ASB$ . Junge  $SC$ , & triangulum  $SBC$ , ob parallelas  $SB$ ,  $Ce$ , æquale erit triangulo  $SBe$ , atq; adeo etiam triangulo  $SAB$ . Simili argumento si  
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Book I, Section II: Motion under centripetal forces.

# Motion under centripetal forces

[ 37 ]

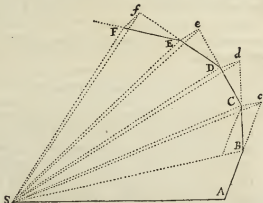
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Book I, Section II: Motion under centripetal forces.

**Proposition I:** Bodies constrained by a central force to orbit a fixed point move in a plane and sweep out equal areas in equal times.

# Motion under centripetal forces

[ 37 ]

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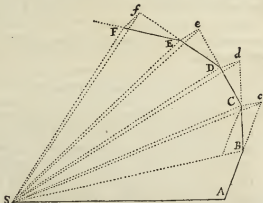
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**Proposition I:** Bodies constrained by a central force to orbit a fixed point move in a plane and sweep out equal areas in equal times.

(Kepler's second law)

# Motion under centripetal forces

[ 37 ]

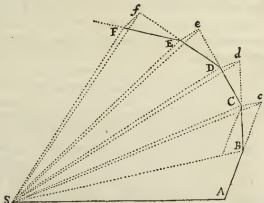
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NB. independent of the 'law of force' involved.



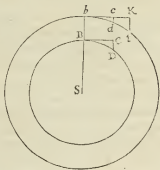
# Book I, Section II: Circular motion

[ 41 ]

Prop. IV. Theor. IV.

Corporum que diversos circulos æquali motu describunt, vires centripetas ad centra eorundem circularum tendere, & esse inter se ut arcuum simul descriptorum quadrata applicata ad circulorum radios.

Corpora  $B, b$  in circumferentiis circulorum  $BD, bd$  gyrantia, simul describant arcus  $BD, bd$ . Quoniam sola vi insita describerent tangentes  $BC, bc$  his arcibus æquales, manifestum est quod vires centripetæ sunt quæ perpetuo retrahunt corpora de tangentibus ad circumferentias circulorum, atq; adeo hæ sunt ad invicem in ratione prima spatiorum nascentium  $CD, cd$ : tendunt vero ad centra circulorum per Theor. II, propterea quod areæ radiis descriptæ ponuntur temporibus proportionales. Fiat figura  $tkb$  figuræ  $DCB$  similis, & per Lemma V, lineola  $CD$  erit ad lineolam  $kt$  ut



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Q. E. D.

Corol. 1. Hinc vires centripetæ sunt ut velocitatum quadrata applicata ad radios circulorum.

Corol. 2. Et reciproce ut quadrata temporum periodicorum appli-

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Book I, Sect. II, Prop. IV: Motion under centripetal forces: motion in a circle.

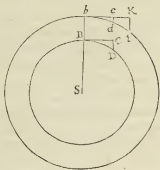
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**Corollary 1:** For motion in a circle centripetal force is proportional to  $\frac{v^2}{r}$ .

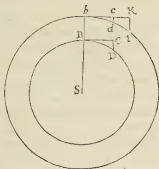
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Book I, Sect. II, Prop. IV: Motion under centripetal forces: motion in a circle.

**Corollary 1:** For motion in a circle centripetal force is proportional to  $\frac{v^2}{r}$ .

**Corollary 6:** For motion in a circle Kepler's third law implies an inverse square law of force.

# Book I, Section III: orbits that are conic sections

[ 50 ]

## S E C T. III.

*De motu Corporum in Conicis Sectionibus eccentricis.*

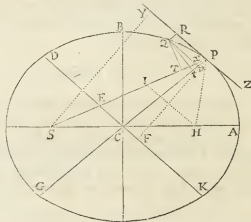
### Prop. XI. Prob. VI.

*Revolvatur corpus in Ellipsi: Requiritur lex vis centripetæ tendentis ad umbilicum Ellipseos.*

Est Ellipseos superioris umbilicus  $S$ . Agatur  $SP$  secans Ellipseos tum diametrum  $DK$  in  $E$ , tum ordinatim applicatam  $Q\psi$  in  $x$ , & compleatur parallelogrammum  $QxPR$ . Patet  $EP$  æ-

qualem esse semi-axi majori  $AC$ , eo quod acta ab altero Ellipseos umbilico  $H$  linea  $HI$  ipsi  $EC$  parallela, (ob æquales  $CS, CH$ ) æquantur  $ES, EI$ , adeo ut  $EP$  semitumma sit ipsarum  $PS, PI$ , id est (ob parallelas  $HI, PR$  & angulos æquales  $IPR, HPZ$ ) ipsorum  $PS, PH$ , quæ

conjunctionem axem totum  $2AC$  adæquant. Ad  $SP$  demittatur perpendicularis  $QT$ , & Ellipseos latere recto principali (seu  $\frac{2BC}{AC}$  quad.) ducto  $L$ , erit  $LxQR$  ad  $LxP\psi$  ut  $QR$  ad  $P\psi$ ; id est ut  $PE$  (seu  $AC$ ) ad  $PC$ : &  $LxP\psi$  ad  $G\psi P$  ut  $L$  ad  $G\psi$ ; &



**Proposition XI:** Motion under centripetal forces: Kepler's First Law (orbit is an ellipse with sun at focus) implies an inverse square law of force.

# Book I, Section III: orbits that are conic sections

[ 50 ]

S E C T. III.

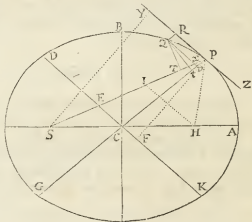
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**Proposition XI:** Motion under centripetal forces: Kepler's First Law (orbit is an ellipse with sun at focus) implies an inverse square law of force.

**Proposition XII:** Motion under centripetal forces: hyperbolic orbit implies an inverse square law of force.

# Book I, Section III: orbits that are conic sections

[ 50 ]

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*De motu Corporum in Conicis Sectionibus eccentricis.*

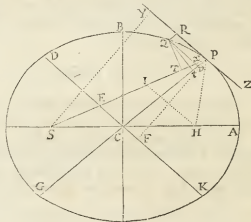
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Est Ellipseos superioris umbilicus  $S$ . Agatur  $SP$  secans Ellipseos tum diametrum  $DK$  in  $E$ , tum ordinatim applicatam  $Q\psi$  in  $x$ , & compleatur parallelogrammum  $QxPR$ . Patet  $EP$  æ-

qualem esse semi-axi majori  $AC$ , eo quod acta ab altero Ellipseos umbilico  $H$  linea  $HI$  ipsi  $EC$  parallela, ( ob æquales  $CS, CH$  ) æquentur  $ES, EI$ , adeo ut  $EP$  sensu summa sit ipsarum  $PS, PI$ , id est ( ob parallelas  $HI, PR$  & angulos æquales  $IPR, HPZ$  ) ipsorum  $PS, PH$ , quæ

conjuncta in axem rotum  $2AC$  adæquant. Ad  $SP$  demittatur perpendicularis  $QT$ , & Ellipseos latere recto principali ( seu  $\frac{2BC}{AC}$  quad. ) disce  $L$ , erit  $LxQR$  ad  $LxP\psi$  ut  $QR$  ad  $P\psi$ ; id est ut  $PE$  ( seu  $AC$  ) ad  $PC$ ; &  $LxP\psi$  ad  $G\psi P$  ut  $L$  ad  $G\psi$ ; &

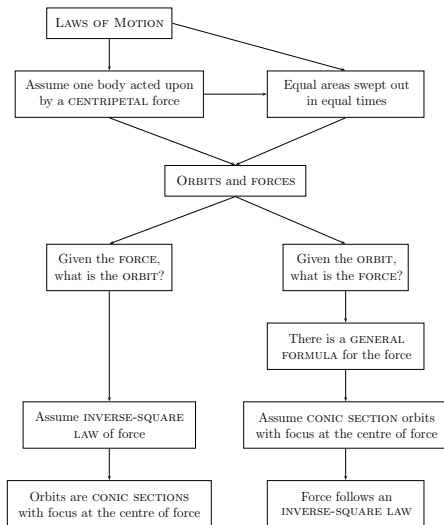


**Proposition XI:** Motion under centripetal forces: Kepler's First Law (orbit is an ellipse with sun at focus) implies an inverse square law of force.

**Proposition XII:** Motion under centripetal forces: hyperbolic orbit implies an inverse square law of force.

**Proposition XIII:** Motion under centripetal forces: parabolic orbit implies an inverse square law of force.

# Book I, Sections II and III summarised



# Book I, later sections

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All treated geometrically

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Book III: The system of the world:

- ▶ Reconciliation of observation and theory
- ▶ Shape of the earth (correct?)
- ▶ Motion of the moon (wrong)
- ▶ Prediction of tides
- ▶ Comets



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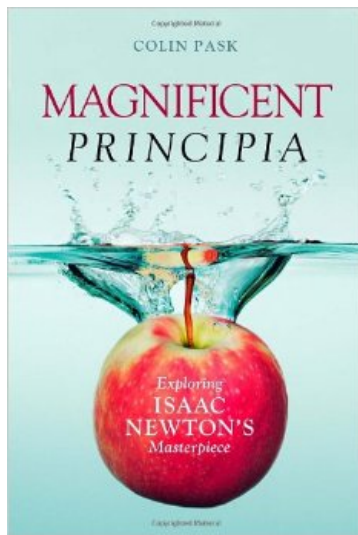
# Influence of the *Principia*

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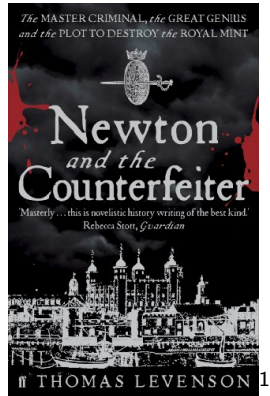
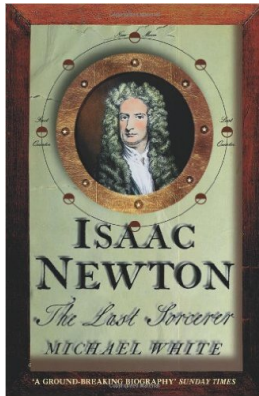
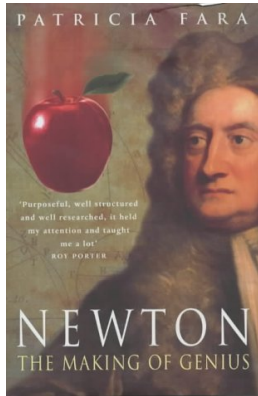
Predictions could be verified by observation and experiment — verified (after some controversy) in the case of the shape of the earth, contradicted in the case of the motion of the moon.

For more on the *Principia*...



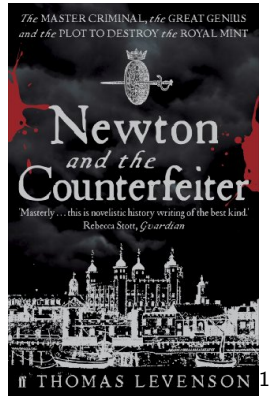
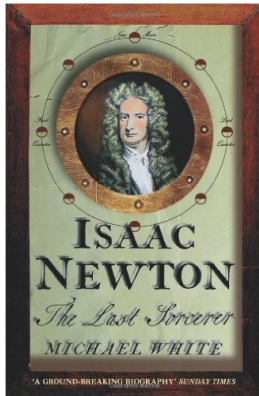
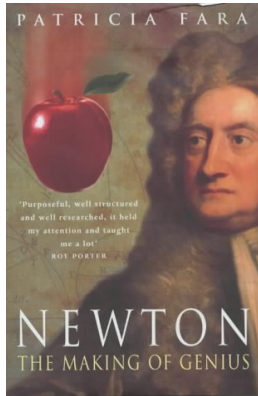
(Colin Pask, *Magnificent Principia*, Prometheus Books, 2013)

## Three (very different) books among many...



<sup>1</sup>Newton: The Making of Genius, by Patricia Fara. Isaac Newton: The Last Sorcerer, by Michael White. Newton and the Counterfeiter, by Thomas Levenson

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And a lecture given at Gresham College:

[www.gresham.ac.uk/lectures-and-events/isaac-newtons-world](http://www.gresham.ac.uk/lectures-and-events/isaac-newtons-world)

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