BO1 History of Mathematics
Lecture VI
Infinite series
Part 1: A non-Western prelude

MT 2021 Week 3

Summary

Part 1

- A non-Western prelude

Part 2

- Newton and the Binomial Theorem
- Other 17th century discoveries
- Ideas of convergence

Part 3

- Much 18th century progress: power series
- Doubts — and more on convergence

The Kerala School

Flourished in Southern India from the 14th to the 16th centuries, working on mathematical and astronomical problems

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Tantrasamgraha (1501)

Completed by Kelallur Nilakantha Somayaji (1444–1544) in 1500; concerns astronomical computations

Keralan series

Infinite series for trigonometric functions appear in Sanskrit verse in an anonymous commentary on the *Tantrasamgraha*, entitled the *Tantrasamgraha-vyakhya*, of c. 1530:

इष्टज्यात्रिज्ययोर्घातात् कोट्याप्तं प्रथमं फलम् ।
ज्यावर्गं गुणकं कृत्वा कोटिवर्गं च हारकम् ॥
प्रथमादिफलेभ्योऽथ नेया फलततिर्मुहुः ।
एकत्र्याद्योज संख्याभिर्भक्तेष्वेतेष्वनुक्रमात् ॥
ओजानां संयुतेस्त्यक्त्वा युग्मयोगं धनुर्भवेत् ।
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Proof supplied by Jyeṣṭhadeva in his *Yuktibhāṣā* (1530)

Keralan series

From the *Tantrasamgraha-vyakhya*:

The product of the given Sine and the radius divided by the Cosine is the first result. From the first, [and then, second, third] etc., results obtain [successively] a sequence of results by taking repeatedly the square of the Sine as the multiplier and the square of the Cosine as the divisor. Divide [the above results] in order by the odd numbers one, three, etc. [to get the full sequence of terms]. From the sum of the odd terms, subtract the sum of the even terms. [The results] become the arc. In this connection, it is laid down that the [Sine of the] arc or [that of] its complement, which ever is smaller, should be taken here [as the given Sine]; otherwise, the terms obtained by the [above] repeated process will not tend to the vanishing magnitude.

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Modern interpretation:

$$R\theta = \frac{R(R\sin\theta)^1}{1(R\cos\theta)^1} - \frac{R(R\sin\theta)^3}{3(R\cos\theta)^3} + \frac{R(R\sin\theta)^5}{5(R\cos\theta)^5} - \cdots$$

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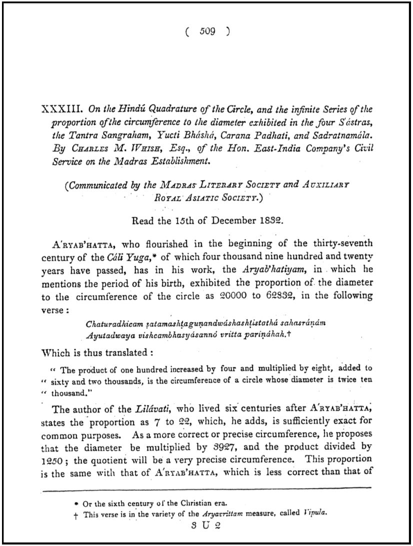
Keralan series

But these results were unknown in the West until the 1830s

As we will see, the series for arctan was reproduced independently in Scotland in the 1670s



(509)

XXXIII. *On the Hindú Quadrature of the Circle, and the infinite Series of the proportion of the circumference to the diameter exhibited in the four S'ástras, the Tantra Sangraham, Yucti Bháshá, Carana Padhati, and Sadratnamála. By* CHARLES M. WHISH, *Esq., of the Hon. East-India Company's Civil Service on the Madras Establishment.*

(Communicated by the MADRAS LITERARY SOCIETY *and* AUXILIARY ROYAL ASIATIC SOCIETY.*)*

Read the 15th of December 1832.

A'RYAB'HATTA, who flourished in the beginning of the thirty-seventh century of the *Cáli Yuga*,* of which four thousand nine hundred and twenty years have passed, has in his work, the *Aryab'hatiyam*, in which he mentions the period of his birth, exhibited the proportion of the diameter to the circumference of the circle as 20000 to 62832, in the following verse:

Chaturadhicam ṣatamashṭaguṇandwáshashṭistathá sahasráṇám
Ayutadwaya vishcambhasyásannó vritta pariṇáhah.†

Which is thus translated:

"The product of one hundred increased by four and multiplied by eight, added to "sixty and two thousands, is the circumference of a circle whose diameter is twice ten "thousand."

The author of the *Liláavati*, who lived six centuries after A'RYAB'HATTA, states the proportion as 7 to 22, which, he adds, is sufficiently exact for common purposes. As a more correct or precise circumference, he proposes that the diameter be multiplied by 3927, and the product divided by 1250; the quotient will be a very precise circumference. This proportion is the same with that of A'RYAB'HATTA, which is less correct than that of

* Or the sixth century of the Christian era.
† This verse is in the variety of the *Aryavrittam* measure, called *Vipula*.

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Keralan Series

A similar struggle between speculation and prudence is provoked by the fascinating conceptual similarities between the Madhava school's methods in infinite series and early modern European infinitesimal calculus techniques. Some scholars have proposed that the former could have been the ultimate source of the latter. In broad outline, this hypothesis suggests that Jesuit missionaries in the vicinity of Cochin in the second half of the sixteenth century sought improved trigonometric and calendric methods in order to solve problems of navigation. Finding the Sine methods described in works of the Kerala school useful for that purpose, they transmitted this knowledge to their correspondents in Europe, whence it was disseminated via informal scholarly networks to the early developers of European calculus methods.

Plofker, Kim. Mathematics in India, Princeton University Press, 2009. Pg. 252.

Keralan Series

Although this idea is intriguing, it does not seem at present to have moved beyond speculation. There are no known records of sixteenth or seventeenth-century Latin translations or summaries of these mathematical texts from Kerala. Nor do the innovators of infinitesimal concepts in European mathematics mention deriving any of them from Indian sources. It is true that historical theories about mathematical transmission in antiquity have sometimes been accepted primarily on the basis of such conceptual similarities rather than of (unavailable) documentary evidence. And the historiographic question thus raised is an interesting one: what are or should be the criteria for accepting a hypothesis of cross-cultural transmission as plausible, and are those criteria culturally dependent?

Plofker, Kim. Mathematics in India, Princeton University Press, 2009. Pg. 252.