BO1 History of Mathematics
Lecture V
Successes of and difficulties with the calculus: the 18th-century beginnings of ‘rigour’
Part 3: Difficulties and responses

MT 2021 Week 3

More problems: infinitely small quantities

Thomas Hobbes, *Six lessons to the Professors of Mathematicks* (1656):

> *The least Altitude is Somewhat or Nothing. If Somewhat, then the first character of your Arithmeticall Progression must not be zero;*
>
> . . .
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> *If Nothing, then your whole figure is without Altitude and consequently your Understanding nought.*

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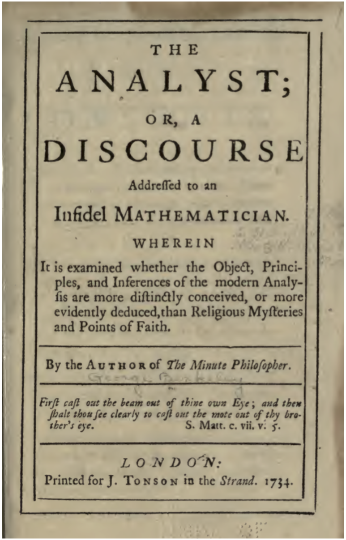
Wallis tried to provide further explanation in his *Due correction for Mr. Hobbes* (1656), but wasn't too concerned by the problems

More problems: infinitely small quantities

George Berkeley (1734)



THE

ANALYST;

OR, A

DISCOURSE

Addreſſed to an

Infidel MATHEMATICIAN.

WHEREIN

It is examined whether the Object, Principles, and Inferences of the modern Analyſis are more diſtinctly conceived, or more evidently deduced, than Religious Myſteries and Points of Faith.

By the AUTHOR of *The Minute Philoſopher*.

Firſt caſt out the beam out of thine own Eye; and then ſhalt thou ſee clearly to caſt out the mote out of thy brother's eye. S. Matt. c. vii. v. 5.

LONDON:

Printed for J. TONSON in the *Strand*. 1734.

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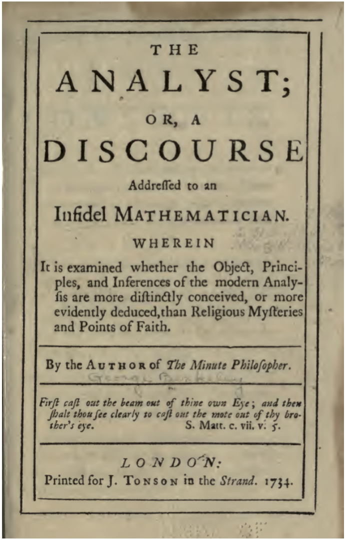
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Qu. 43: Whether an algebraist, fluxionist, geometrician, or demonstrator of any kind can expect indulgence for obscure principles or incorrect reasoning? And whether an algebraical note or species can at the end of a process be interpreted in a sense which could not have been substituted for it at the beginning?

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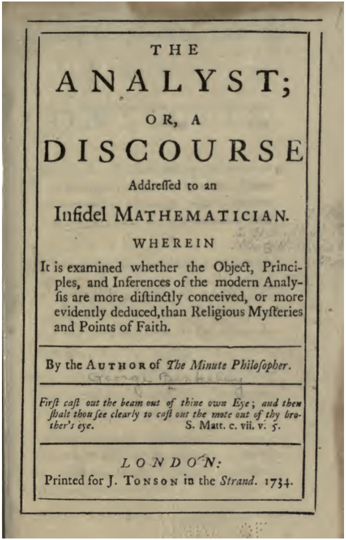
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Qu. 45: Whether, although geometry be a science, and algebra allowed to be a science, and the analytical a most excellent method, in the application nevertheless of the analysis to geometry, men may not have admitted false principles and wrong methods of reasoning?

Some responses to the difficulties

Guillaume Marquis de l'Hôpital, *Analyse des infiniment petits* (1696)

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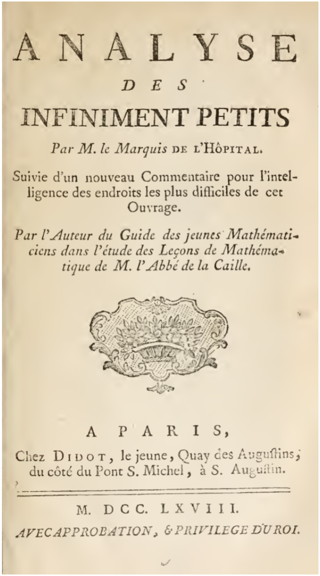
Joseph-Louis Lagrange, *Théorie des fonctions analytiques* (1797)

Responses to the difficulties: l'Hôpital



ANALYSE
DES
INFINIMENT PETITS

Par M. le Marquis DE L'HÔPITAL.

Suivie d'un nouveau Commentaire pour l'intelligence des endroits les plus difficiles de cet Ouvrage.

Par l'Auteur du Guide des jeunes Mathématiciens dans l'étude des Leçons de Mathématique de M. l'Abbé de la Caille.

A PARIS,

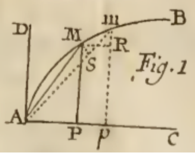
Chez DIDOT, le jeune, Quay des Auguſtins; du côté du Pont S. Michel, à S. Auguſtin.

M. DCC. LXVIII.

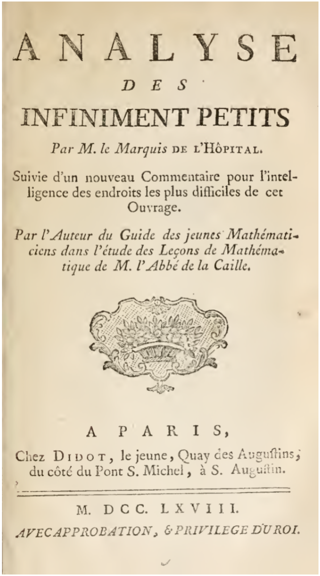
AVEC APPROBATION, & PRIVILEGE DU ROI.

Guillaume, Marquis de l'Hôpital (1696)

Definition. The infinitely small part whereby a variable quantity is continually increased or decreased, is called the differential of that quantity.



Responses to the difficulties: l'Hôpital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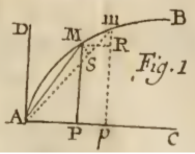
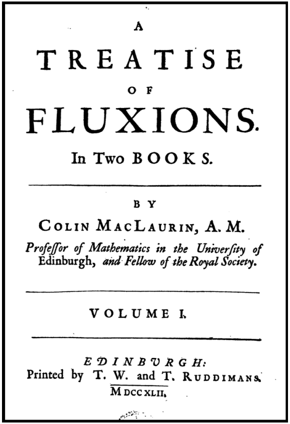


Postulate. Grant that two quantities whose difference is an infinitely small quantity may be taken (or used) indifferently for each other: or (which is the same thing) that a quantity which is increased or decreased only by an infinitely small quantity may be considered as remaining the same.

Responses to the difficulties: Colin Maclaurin (1742)



A
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In Two BOOKS.

BY
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Professor of Mathematics in the University of Edinburgh, *and Fellow of the Royal Society.*

VOLUME I.

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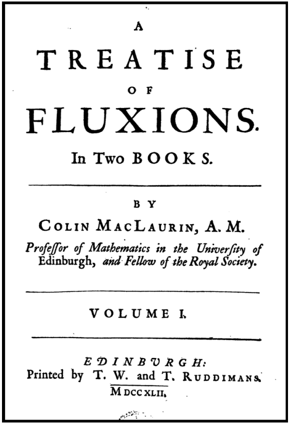

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  derive a contradiction from the assumption that $a > b$;
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  then it must be the case that $a = b$).

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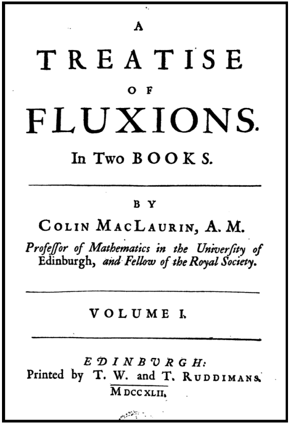
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Grabiner, J. (2010). *A historian looks back: The calculus as algebra and selected writings*. Washington, D.C.: MAA. (Part II, Chapter 7)

Responses to the difficulties: Maria Agnesi (1748)


INSTITUZIONI
ANALITICHE
AD USO
DELLA GIOVENTU' ITALIANA
DI D.NA MARIA GAETANA
AGNESI
MILANESE
Dell' Accademia delle Scienze di Bologna.
TOMO I.

IN MILANO, MDCCXLVIII.
NELLA REGIA-DUCAL CORTE.
CON LICENZA DE' SUPERIORI.

since the publication of the aforementioned book, many important and useful discoveries have been made by many ingenious writers... therefore, to save students the trouble of seeking for these improvements, and newly-invented methods, in their several authors, I was persuaded that a new Digest of Analytical Principles might be useful and acceptable.

Maria Agnesi (tr. John Colson), *Analytical Institutions* (1801). Pg xxii.

Read at archive.org

Responses to the difficulties: Maria Agnesi (1748)

if indeed they [ladies of Britain] can be prevailed upon by his persuasion and encouragement, to show to the world, as they easily might, that they are not to be excelled by any foreign Ladies whatever, in any valuable accomplishment.

Maria Agnesi (tr. John Colson), *Analytical Institutions* (1801). Pg vi.

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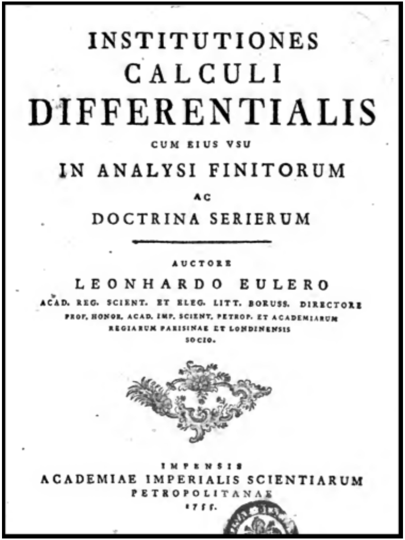
Responses to the difficulties: Euler



INSTITUTIONES
CALCULI
DIFFERENTIALIS
CUM EIUS VSU
IN ANALYSI FINITORUM
AC
DOCTRINA SERIERUM

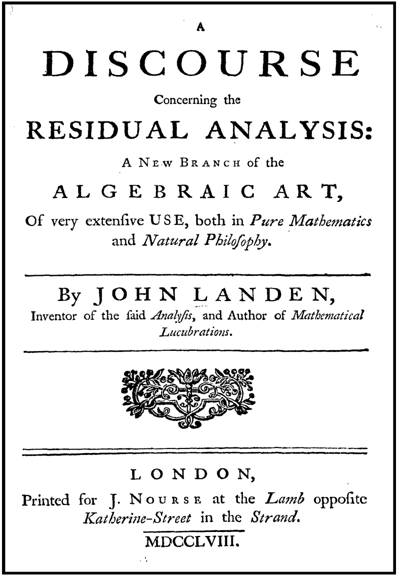
AUCTORE
LEONHARDO EULERO
ACAD. REG. SCIENT. ET ELEG. LITT. BORUSS. DIRECTORE
PROF. HONOR. ACAD. IMP. SCIENT. PETROP. ET ACADEMIARUM
REGIARUM PARISINAE ET LONDINENSIS
SOCIO.

IMPENSIS
ACADEMIAE IMPERIALIS SCIENTIARUM
PETROPOLITANAE
1755.

Leonard Euler (1755):

> *An infinitely small quantity is nothing other than a vanishing quantity, and is therefore really equal to 0.*
>
> ...
>
> *If there occur different infinitely small quantities dx and dy, although both are equal to 0, nevertheless their ratio is not constant.*

Responses to the difficulties: Landen



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MDCCLVIII.

John Landen (1758)

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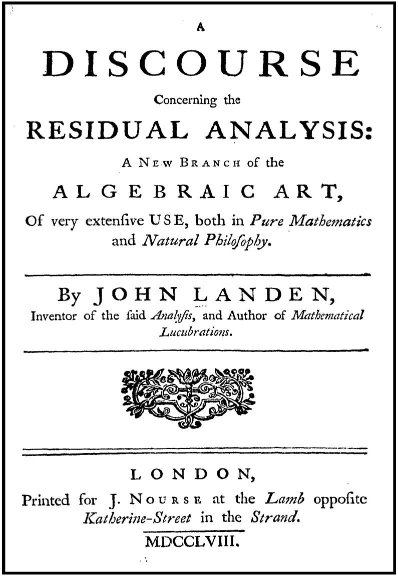

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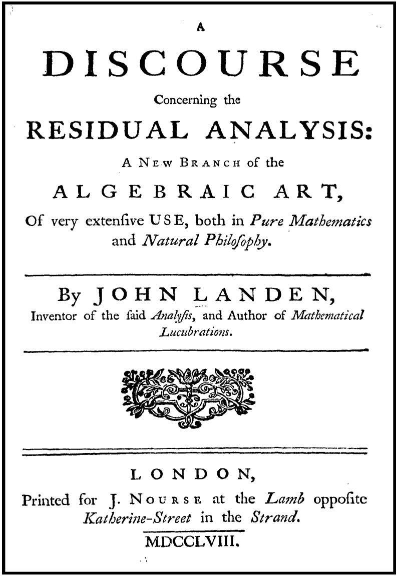
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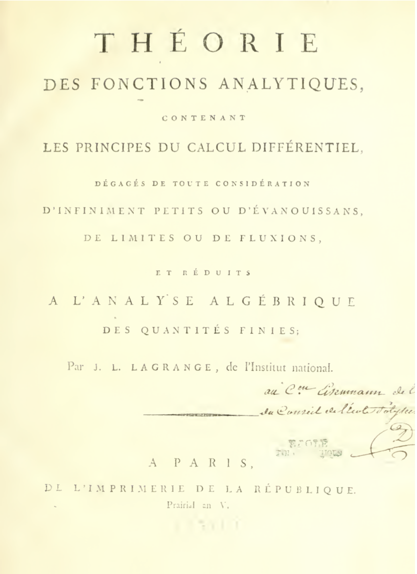
MDCCLVIII.

John Landen (1758)

- ‘Fluxions are not immediately applicable to algebraic quantities …’
- attempted a purely algebraic development of calculus
- Elected FRS 1765

See Royal Society Election Certificates Online here.

Responses to the difficulties: Lagrange



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DÉGAGÉS DE TOUTE CONSIDÉRATION

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DE LIMITES OU DE FLUXIONS,

ET RÉDUITS

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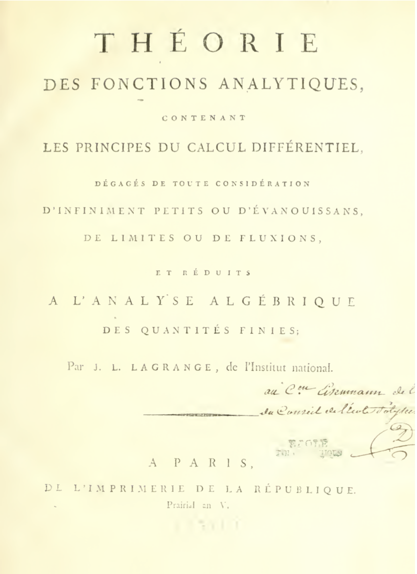

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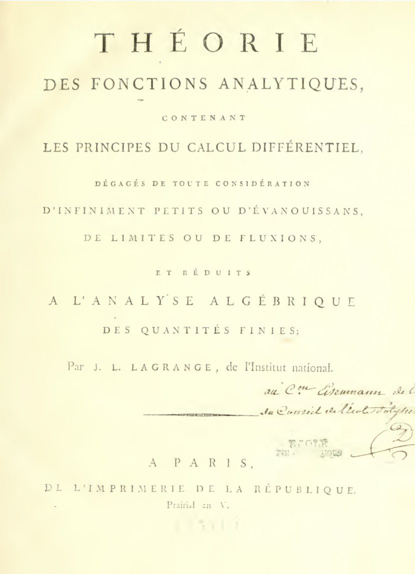
Another attempt to avoid ‘infinitely small quantities’

Responses to the difficulties: Lagrange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Another attempt to avoid 'infinitely small quantities'

(by taking functions to be defined by power-series expansions)

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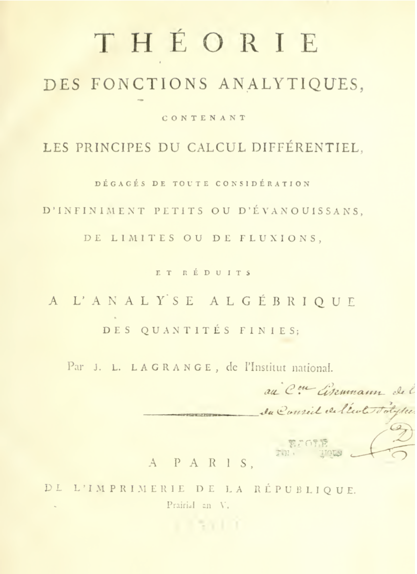
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See *Mathematics emerging*, §14.1.2.

Responses to the difficulties: Lagrange

..., et ainsi de suite.

Donc, substituant ces valeurs dans le développement de la fonction $f(x+i)$, on aura

$$f(x+i) = fx + f'xi + \frac{f''x}{2}i^2 + \frac{f'''x}{2\cdot 3}i^3 + \frac{f^{\text{IV}}x}{2\cdot 3\cdot 4}i^4 + \&c.$$

Cette nouvelle expression a l'avantage de faire voir comment les termes de la série dépendent les uns des autres, et sur-tout comment, lorsqu'on sait former la première fonction dérivée d'une fonction primitive quelconque, on peut former toutes les fonctions dérivées que la série renferme.

17. Nous appellerons la fonction fx, *fonction primitive*, par rapport aux fonctions $f'x$, $f''x$, &c. qui en dérivent, et nous appellerons celles-ci, *fonctions dérivées*, par rapport à celle-là. Nous nommerons de plus la première

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Finally, note the increasing ‘professionalisation’ of mathematics in the eighteenth century:

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  - Lagrange followed Euler to Berlin, later went to Paris;
- each Academy had its own ‘Mémoires’ or ‘Transactions’ enabling wider (and sometimes faster) circulation of new ideas.