BO1 History of Mathematics
Lecture V
Successes of and difficulties with the calculus: the 18th-century beginnings of 'rigour'
Part 1: Publication, acceptance, and successes

MT 2021 Week 3

Summary

Part 1

- Publication and acceptance of the calculus
- Some successes of the calculus

Part 2

- Functions

Part 3

- Problems with the calculus
- Some responses: beginnings of ‘rigour’ in Analysis

Reminder: a comparison from lecture IV

Newton (1664–65):	Leibniz (1673–76):
rules for quadrature	rules for quadrature
rules for tangents	rules for tangents
'fundamental theorem'	'fundamental theorem'
dot notation	differential notation
physical intuition:	algebraic intuition
rates of change	rules and procedures
PROBLEM:	PROBLEM:
vanishing quantities o	vanishing quantities du, dv, ...

Newton's publication (or not) of his calculus

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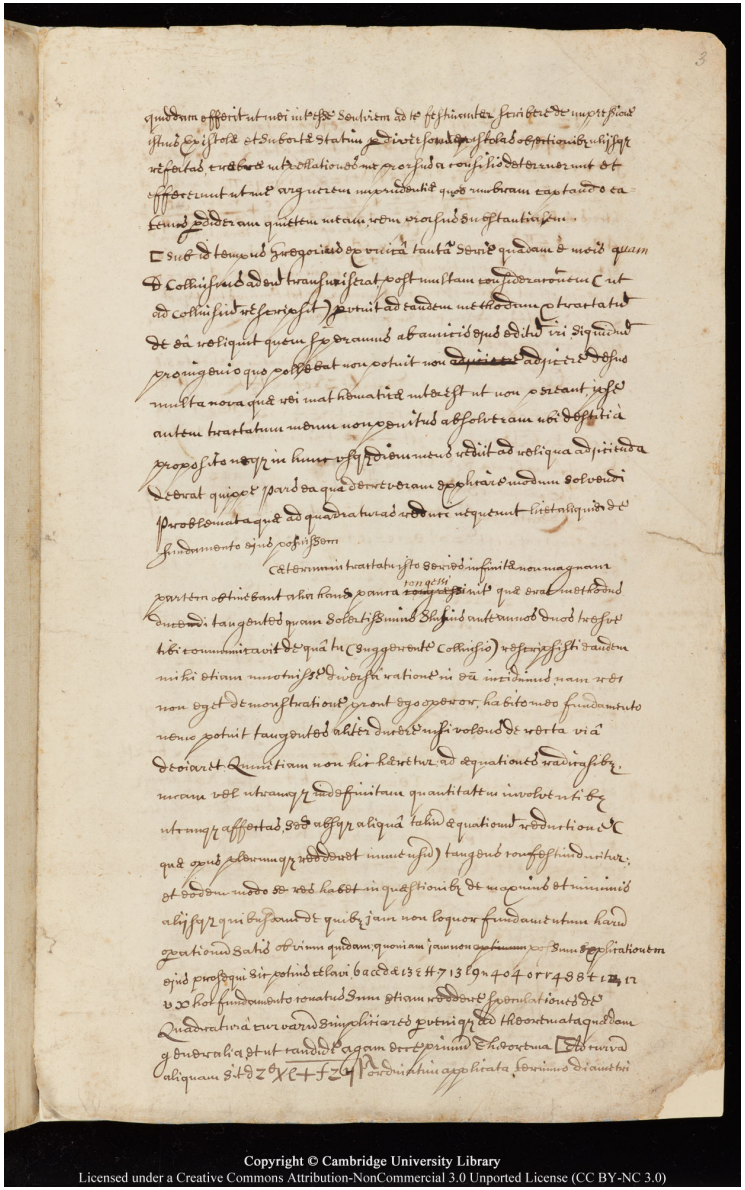
1704: 'Treatise of quadrature' appended to published *Opticks*

Newton's coded message



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Letter from Isaac Newton to Henry Oldenburg, 24 October 1676 ('Epistola posterior')

"The foundation of these operations is evident enough, in fact; but because I cannot proceed with the explanation of it now, I have preferred to conceal it thus: 6*accdae*13*eff*7*i*3*l*9*n*4*o*4*qrr*4*s*8*t*12*vx*."

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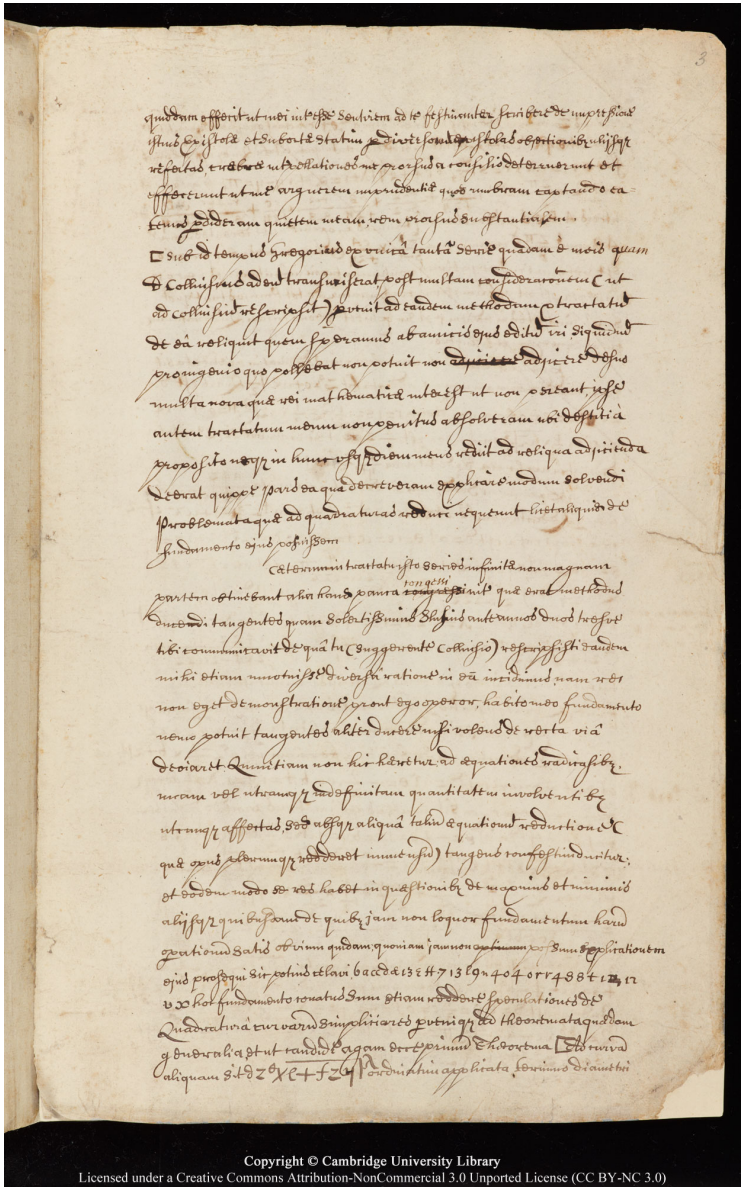

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"Data aequatione quotcunque fluentes quantitates involvente, fluxiones invenire: et vice versa."

= "Given an equation involving any number of fluent quantities, to find the fluxions: and vice versa."

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1696: Exposition by L'Hôpital based on teachings of Johann Bernoulli

Challenge problems

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1687: Isochrone — curve of uniform descent
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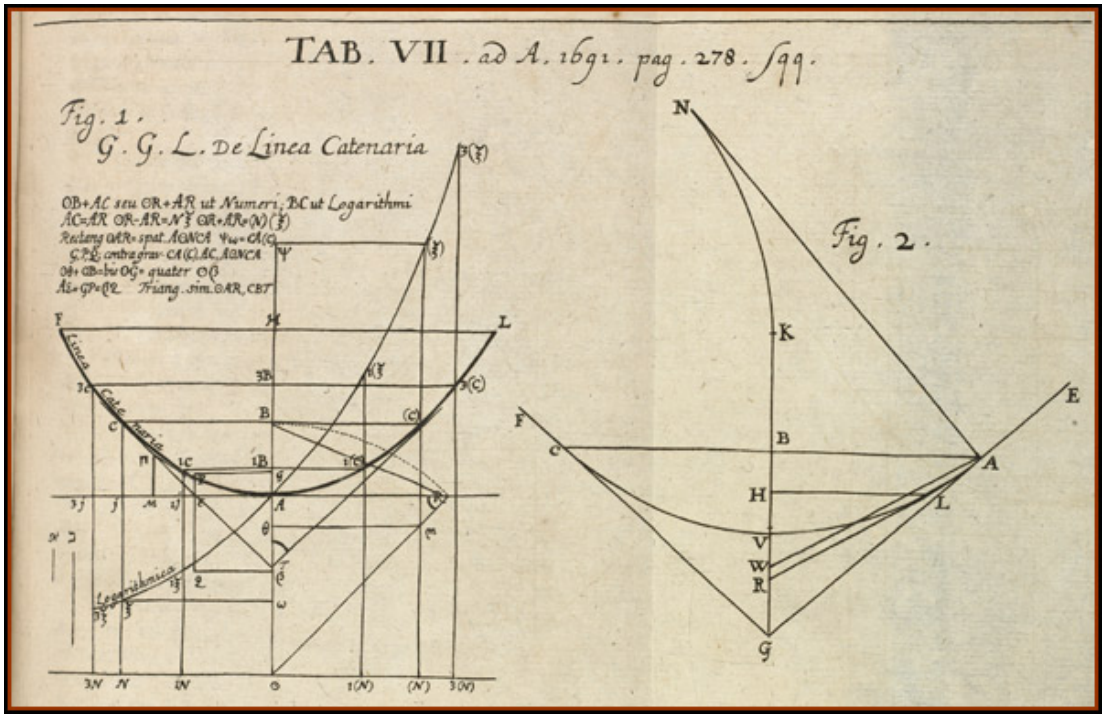
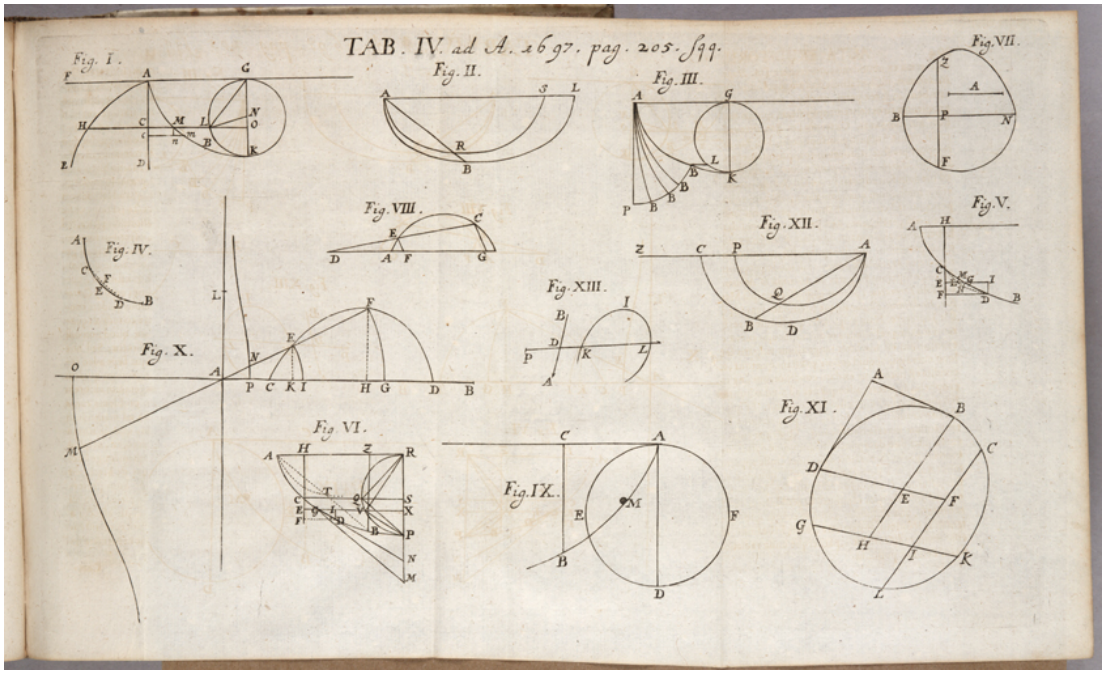
1691: Catenary — curve of a hanging chain
(posed by Jacob Bernoulli; solved by Johann Bernoulli, Huygens, Leibniz)



Leibniz' and Huygens' solutions, *Acta eruditorum*, 1691.

Challenge problems



Solutions by Johann & Jacob Bernoulli, l'Hospital, and Newton, *Acta eruditorum*, 1696.

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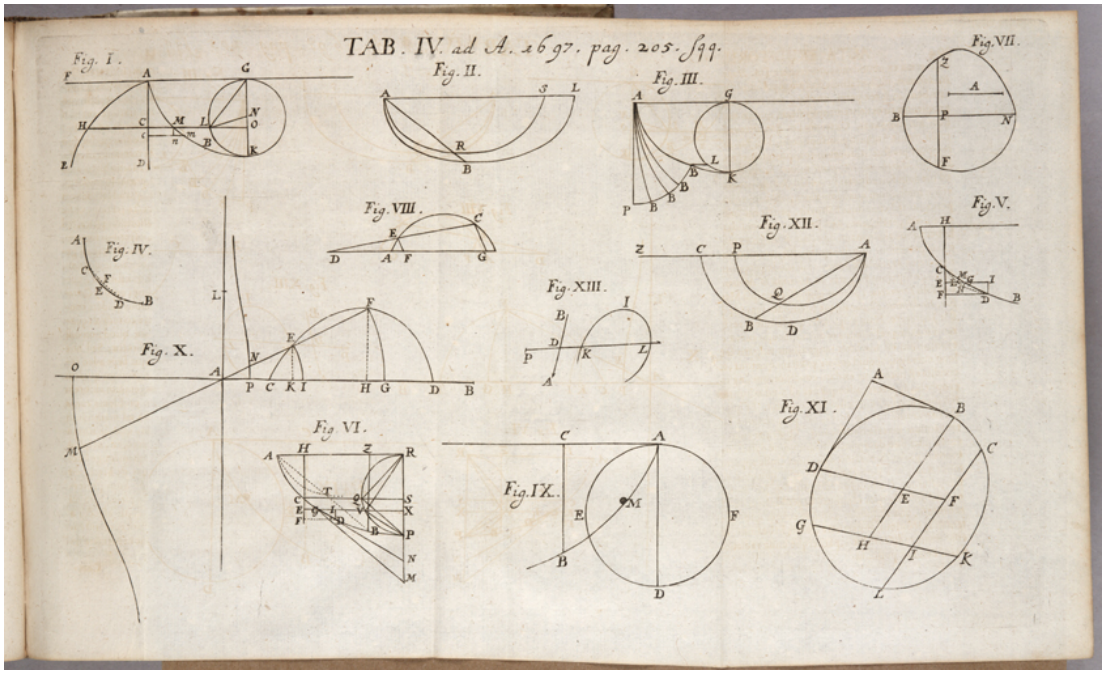

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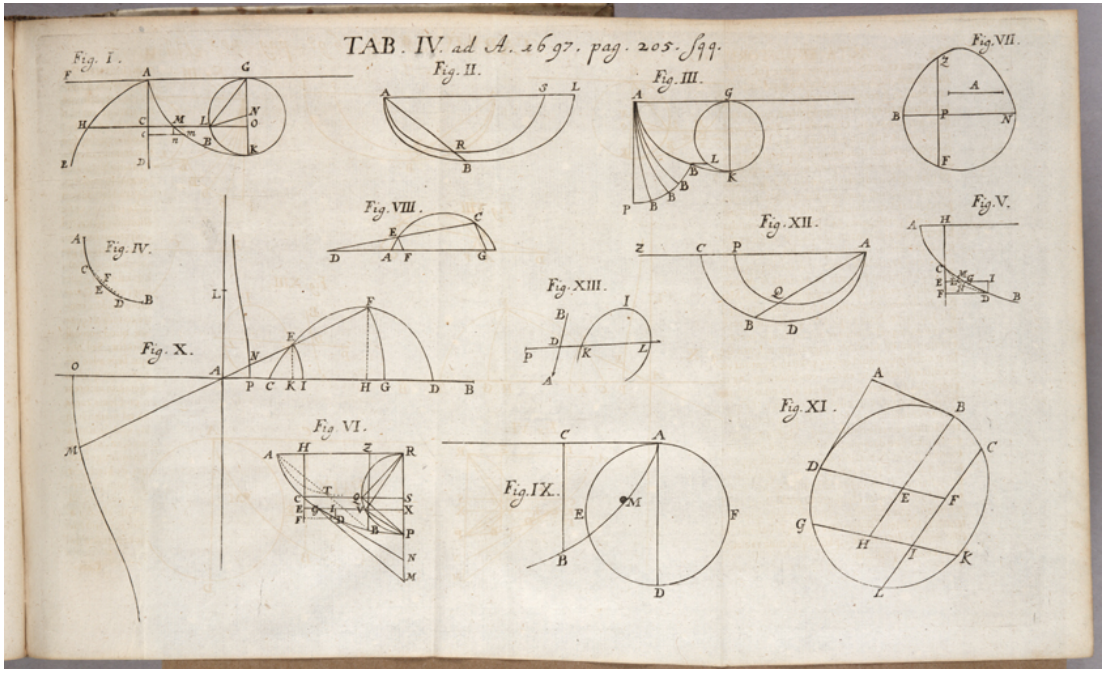

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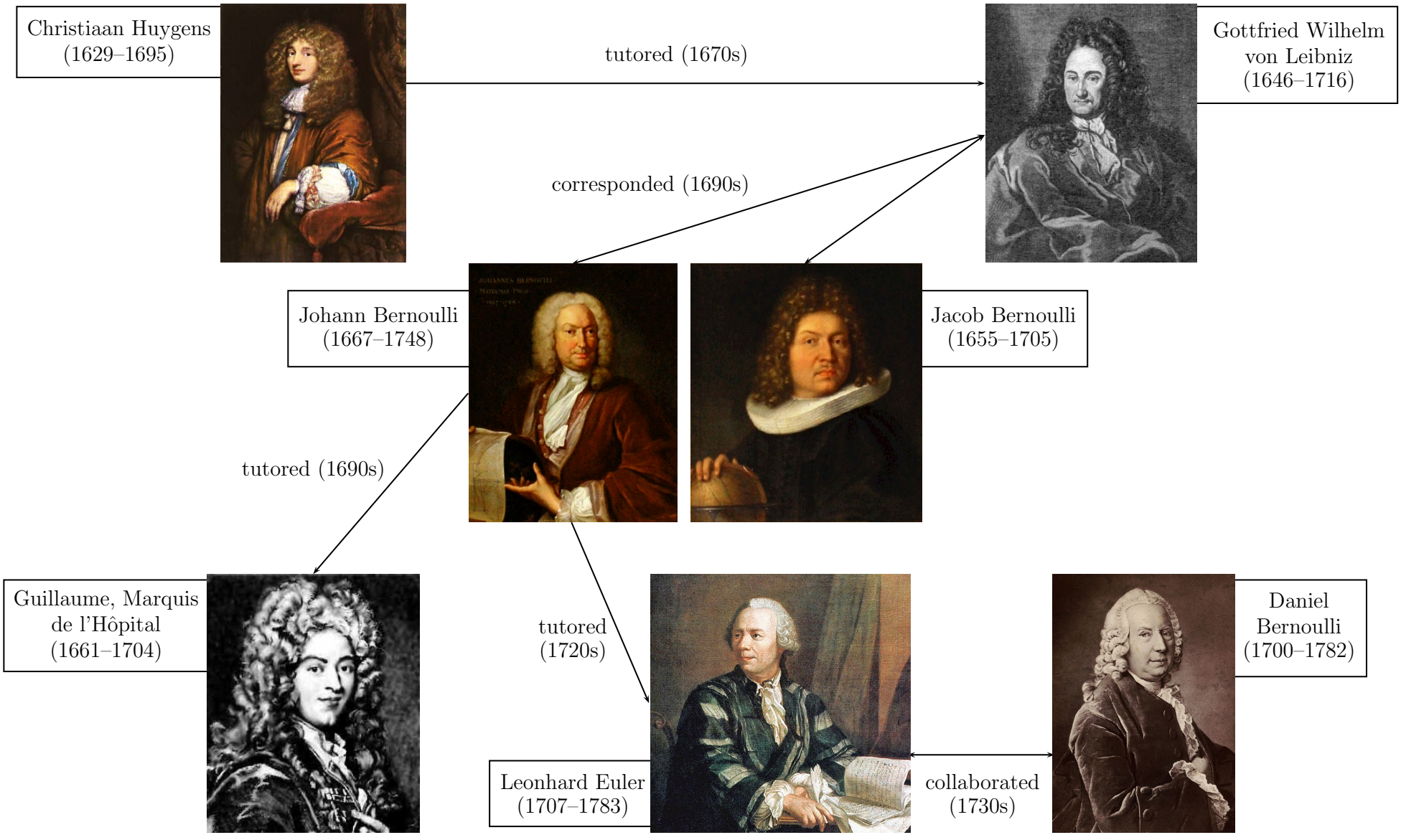
And many others

People and connections



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- consolidate and validate Leibnizian calculus
- introduce new questions about ‘functions’, ‘differentiability’, ‘continuity’, ...