BO1 History of Mathematics Lecture VI Infinite series Part 2: The 17th century

MT 2021 Week 3

Infinite series 1600–1900: an overview

Lecture VI:

- mid–late 17th century: many discoveries
- early 18th century: much progress
- later 18th century: doubts and questions

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Infinite series 1600–1900: an overview

Lecture VI:

- mid–late 17th century: many discoveries
- early 18th century: much progress
- later 18th century: doubts and questions

Lecture VII:

- early 19th century: Fourier series
- early 19th century: convergence better understood

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CUL Add. MS 3958.3, f. 72

(See lecture IV)

If lab is an Hyperfile; eds, ch its symplotes, also. le + 10x2 + 10x3 + 5x4 + x5 %). The. times proceeding this prographion. * =adel. TR x+ 3×× +x3, x+2×× + 3×3 + ×+ x++×× + & first avia is also inserted. The composition 1. Acducid from hence; vire: The same of igure above it is equall to y' . By wel table it may appeare of of y' Hyperbola adeb -x" +x7 -x8 + x9 - ×10 8te Suppose of adek abe a civele age a Parafola 81 = x . Mall fr=1= y lines fr. be progression in with 1 VI-XX. 1-XX. 1-XX. 1-XX. 1-XX. +X4. 1-XX.+X4. 1-3xx + 3xt =-x6. Ve. Then will this avias fire, bars, gade, in this propristion. x. *. x- *** *. x- =x3+= xr. + * x-4x3+6x5-4x7+x9 inserted. The proper above it 12. 6. after it save one. alles & per ion : are of this 22. 15. maconvall progressi formes 15. 60 ard. Tert. O. Tert. O. - Tert. O. Mar. 1. C. Ter Cond. and Asterset. intermidiate termes may be casily performed. The are ab 4th Collame 1. t. - + - + + (well propression may 1X1X-1X3X-2727 2X11413 X15 () Whereby is may appeare of what Since $\Im_{L=X}$ is, ϑ_{Y}^{L} avec abid is $x - \frac{\pi}{21} - \frac{\pi}{21} - \frac{\pi}{112} - \frac{\pi}{112} - \frac{\pi}{112} - \frac{\pi}{1112} - \frac{\pi}{1112} - \frac{\pi}{1112} - \frac{\pi}{1112}$ area aff is 22+25+25+12 (r.) Whereby also ye area & angle and may be found this. It arres of , all , agd , all ye are in this progression & N. 2+ ** **** - 2 - 2 + 5 × 7 + 5 × 7 + 5 + 8 - 18 × 5 * 40×7 - 12×2. We die in the following Track + 35-29 + 632 " . ye. And by this means having yt area abd, O. Tr. A. (with at anythe and gives) for since of at anythe Did may the friend 0.- 11 0 Good: If Wax & Warmer = 16. y * ekig an Hyputhia. +

Recall: Newton's integration of $(1 + x)^{-1}$

	$(1 + x)^{-1}$	$(1 + x)^0$	$(1 + x)^1$	$(1 + x)^2$	$(1 + x)^3$	$(1 + x)^4$	
x	1	1	1	1 1		1	
$\frac{x^2}{2}$	-1	0	1	2	3	4	
$\frac{x^3}{3}$	1	0	0	1	3	6	
$\frac{x^4}{4}$	-1	0	0	0	1	4	
$\frac{x^5}{5}$	1	0	0	0	0	1	
:	-						

The entry in the row labelled $\frac{x^m}{m}$ and the column labelled $(1 + x)^n$ is the coefficient of $\frac{x^m}{m}$ in $\int (1 + x)^n dx$. (NB. Newton did not use the notation $\int (1 + x)^n dx$.)

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In fact, this method extends easily to any integer n

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Newton's explanation:

The property of which table is that the sum of any figure and the figure above it is equal to the figure next after it save one. Also the numerall progressions are of these forms.

а	а	а	а	
b	a + b	2a + b	3a + b	
с	b + c	a+2b+c	3a+3b+c	&с.
d	c + d	b+2c+d	a+3b+3c+d	
е	d + e	c + 2d + e	b+3c+3d+e	

(See: *Mathematics emerging*, §8.1.1.)

	$(1-x^2)^{-1}$	$(1-x^2)^{-\frac{1}{2}}$	$(1-x^2)^0$	$(1-x^2)^{\frac{1}{2}}$	$(1 - x^2)^1$	$(1-x^2)^{\frac{3}{2}}$	$(1 - x^2)^2$	
x	1	1	1	1	1	1	1	
$-\frac{x^3}{3}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	
$\frac{x^5}{5}$	1	$\frac{3}{8}$	0	$-\frac{1}{8}$	0	$\frac{3}{8}$	1	
$-\frac{x^7}{7}$	-1	$-\frac{5}{16}$	0	$\frac{3}{48}$	0	$-\frac{1}{16}$	0	
$\frac{x^9}{9}$	1	35 128	0	$-\frac{15}{384}$	0	$\frac{3}{128}$	0	
:				:				·

The entry in the row labelled $\pm \frac{x^m}{m}$ and the column labelled $(1 - x^2)^n$ is the coefficient of $\pm \frac{x^m}{m}$ in $\int (1 - x^2)^n dx$.

(NB: possible slips in the last two rows of the original table)

Can fill in some initial values by other methods

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Newton applied the formula

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$$

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to fractional n,

Can fill in some initial values by other methods

Newton applied the formula

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$$

to fractional n, so that

$$\binom{1/2}{1} = \frac{1}{2}, \quad \binom{1/2}{2} = \frac{1/2(1/2 - 1)}{2!} = -\frac{1}{8}$$

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and so on

Newton went on to extend this method to other fractional powers, and also to $(a + bx)^n$, thereby convincing himself of the truth of the general binomial theorem

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On Newton and the binomial theorem, see https://www.youtube.com/watch?v=xv_PWwdDWDk

One more table

The table at the bottom of the page gives the interpolations for $(1 + x)^n$ for half-integer *n*

If lab is an Hyperbola eds, ch its symplotes also. 10x2 + 10x3 + rx4 + xr &c). The. proceediation this prographion. 3×× +x3, x+2×× + 3×3 + ×4, x+4×× + first are is also inserter. The composition 1. Aducid from Rence; vir: The same igure above it is equall to y By well table it may appeare of y' Hyperbola adeb -x + x7 - x8 + x9 - x10 8te Suppose of adek abe a civele age a Parafola 81 = x . Mall fr=1= y lines fr. By 1-xx /1-xx. 1-+xx+x4. 1-+xx+x4/1-xx. 1.VI-XX. 1-XX. Then will this areas fair, Gade, gade, preserigion . x. ¥ . x- *** +. x- =x3+= xr. + above it . 6. after it save one. alles of the nacoverale progressions are of this 15. formes 15. 60 ard. 0. 1024. 0. 111. 1. C. 242 C+22+2. 8+2c+31+2. Tert. ana ap intermediate termes may bee sayily performed. The 4th Collame 1. t. - t. I see (well proprission may x1x-1x3x-px7x-9x11413x15 (fc) WREVERy if may appears y, what x2x4 x6 x8 x10x12x14x16x16 sine & = x is, \$ y & area abid is x - \$ - x - - x - - x - 112 Se.) Whereby also ye area is angle and may see found this. It arres of , all, agd, all ye are in this progression & N. 30 2x1 + x + 23 - 18 x5 + 40 x7 - 12 x2 . Ste cits in the following Tack may see pereceived of all = {x+tx x + 1 x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x + 35-29 + 632 H. No. And by this means having y' area ald. to be we at ugh all gives the sine of at angle all may be friend 0. - 11 0 Boyel of B= x & Dyram = 16. y and a Hyperbola. +

By further interpolations and integrations (based on strong geometric intuition) Newton found further series for:

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(1 + x)^{$$p/q$$}

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log, antilog

sin, tan, ... (NB: cosine was not yet much in use)

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▶ arcsin, arctan, ...

(See: Mathematics emerging, §§8.1.2–8.1.3.)

Newton on the move from finite to infinite series

And whatever common analysis performs by equations made up of a finite number of terms (whenever it may be possible), this method may always perform by infinite equations: in consequence, I have never hesitated to bestow on it also the name of analysis.

(*De analysi*, 1669; Derek T. Whiteside, *The mathematical papers of Isaac Newton*, CUP, 1967–1981, vol. II, p. 241)

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Other 17th-century discoveries (1a)

Brouncker, c. 1655, published 1668: area under the hyperbola given by $\frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + \cdots$



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Other 17th-century discoveries (1b)

$$E dCDE = \frac{1}{2x_3x_4} + \frac{1}{4x_5x_6} + \frac{1}{6x_7x_8} + \frac{1}{9x_{10}} & c.$$

$$E dCDE = \frac{1}{2x_3x_4} + \frac{1}{4x_5x_6} + \frac{1}{6x_7x_8} + \frac{1}{8x_9x_{10}} & c.$$

$$E dCyE = \frac{1}{2x_3x_4} + \frac{1}{4x_5x_6} + \frac{1}{6x_7x_8} + \frac{1}{8x_9x_{10}} & c.$$

(647)



And that therefore in the first feries balf the first term is greater than the fum of the two next, and half this fum of the fecond and third greater than the fum of the four next, and half the fum of those four greater than the fum of the next eight, \mathcal{C}_c , in infinitum. For $\frac{1}{2} dD = br + bn$; but bn > fG, therefore $\frac{1}{2} dD > br + fG$, \mathcal{C}_c . And in the fecond feries half the first term is lefs then the fum of the two next, and half this fum lefs then the fum of the four next, \mathcal{C}_c in infinitum.

That the first fories are the even terms, viz. the a^{4} , b^{6} , b^{6} , b^{7} , 10^{6} , c^{2} , and the fecond, the odd, viz. the 1^{4} , 3^{4} , 5^{60} , 7^{60} , 9^{60} , 6^{60} , 8^{70} , 10^{60} , c^{2} , and the fecond, the odd, viz. the 1^{4} , 3^{4} , 5^{60} , 7^{60} , 9^{60} , c^{60} , 8^{70} , 10^{60} , c^{2} , 10^{10} , c^{2} , c^{2} , 10^{10} , c^{2}

ning, and $\frac{1}{a - 1}$ the fum of the reft to the end.

That $\frac{1}{2}$ of the first terme in the *third* feries is lefs than the fum of the two next, and a quarter of this fum, lefs than the fum of the four next, and one fourth of this last fum lefs than the next eight, I thus demonstrate.

Let a the 3" or laft number of any term of the first Column, viz: of Divifors,

Other 17th-century discoveries (2)

Mercator's series (1668), found by long division:

$$\frac{1}{1+a} = 1 - a + aa - a^3 + a^4 (\&c.)$$

Gives rise to series for log



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Other 17th-century discoveries (3)



James Gregory (1671): ► general binomial expansion



Other 17th-century discoveries (3)



James Gregory (1671):

- general binomial expansion
- series for tan, sec, and others, including

$$\theta = \tan \theta - \frac{1}{2} \tan^3 \theta + \frac{1}{5} \tan^5 \theta - \cdots$$

for
$$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

Other 17th-century discoveries (3)



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For $-rac{\pi}{4} \le heta \le rac{\pi}{4}$

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Gregory to Collins, 23rd November 1670:

I suppose these series I send here enclosed, may have some affinity with those inventions you advertise me that Mr. Newton had discovered.

(On Gregory's work, see: *Mathematics emerging*, §8.1.4.)

Other 17th-century discoveries (4)

Gottfried Wilhelm Leibniz (1675):

The area of a circle with unit diameter is given by

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \&c.$$

Other 17th-century discoveries (4)

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The area of a circle with unit diameter is given by

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \&c.$$

The error in the sum is successively less than $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc.

Therefore the series as a whole contains all approximations at once, or values greater than correct and less than correct: for according to how far it is understood to be continued, the error will be smaller than a given fraction, and therefore also less than any given quantity. Therefore the series as a whole expresses the exact value.

(See: *Mathematics emerging*, §8.3.)

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

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(Determined that

$$\Box>\sqrt{\frac{3}{2}},\quad \Box<\frac{3}{2}\sqrt{\frac{3}{4}},$$

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(Determined that

$$\Box > \sqrt{\frac{3}{2}}, \quad \Box < \frac{3}{2}\sqrt{\frac{3}{4}}, \quad \Box > \left(\frac{3\times3}{2\times4}\right)\sqrt{\frac{5}{4}},$$

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and so on)

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Brouncker (1668): grouping of terms

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Brouncker (1668): grouping of terms

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Leibniz (1675): 'alternating' series
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Power series (infinite polynomials):

enabled term-by-term integration for difficult quadratures;

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helped establish sine, log, ... as 'functions' (transcendental);

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Power series (infinite polynomials):

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encouraged a move from geometric to algebraic descriptions;

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Power series rank with calculus as a major advance of the 17th century

Calculus and series combined

Newton's treatise of 1671, published 1736

METHOD of FLUXIONS

AND

INFINITE SERIES;

WITH ITS

Application to the Geometry of CURVE-LINES.

By the INVENTOR Sir ISAAC NEWTON, Kr. Late Prefident of the Royal Society.

Tranflated from the AUTHOR'S LATIN ORIGINAL not yet made publick.

To which is fubjoin'd, A PERPETUAL COMMENT upon the whole Work,

Confiling of ANNOTATIONS, ILLUSTRATIONS, and SUPPLEMENTS,

In order to make this Treatine A compleat Inflitution for the use of LEARNERS.

By JOHN COLSON, M.A. and F.R.S. Mafter of Sir Joseph Williamsen's free Mathematical-School at Rockoffer.

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