How to Write a [C] Special Topic

M.Sc. in Mathematical Modelling & Scientific Computing,
Additional Skills

26th November 2021

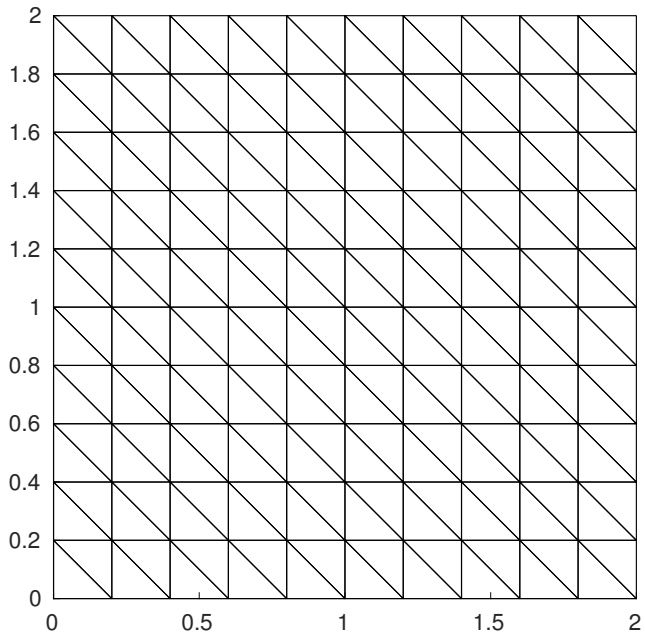
Key Points

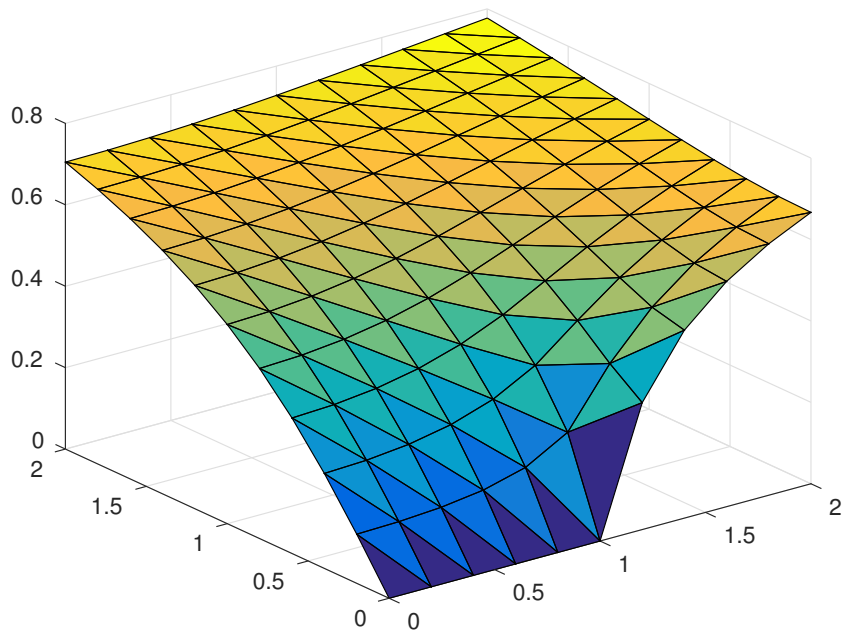
- go beyond the lectures, either by looking at more advanced numerical methods or implementing methods on problems beyond the most basic;
- ideally describe the method (or a range of methods), its advantages and disadvantages compared to other methods, state necessary theorems, show some examples;
- make clear what parameter values are used in your calculations (if appropriate) and the effect of these parameters;
- compare numerical results to theory and explain any discrepancies;
- summarise work at end and cite sources throughout.

Example: Finite Element Methods for PDEs

Basic idea of the course: the theory of finite element methods is described for (mostly elliptic) PDEs. The idea is to split the solution domain into a set of "finite elements" — these are often triangles in 2D — and to approximate the solution to the weak formulation of the problem using a continuous function which is a polynomial in each element.

Example





Finite Element Special Topic

In the course, results are proved which relate $\|u - u_h\|_{H^1(\Omega)}$ and $\|u - u_h\|_{L_2(\Omega)}$ to the mesh size and the degree of the approximating polynomial (here u_h is the finite element solution).

Possible special topic idea: solve Poisson's equation on an arbitrary (non-rectangular) polygonal domain $\Omega \subset \mathbb{R}^2$. This will require you to triangulate the domain and to use quadrature.

(Note that for this project the challenges lie in the coding but the mathematics still needs to be clearly and accurately explained.)

Title Page

The title page should consist of the title of the special topic, the name of the lecture course and your candidate number (NOT your name). You may also add a university crest. If this is all that is on the title page it does not contribute to the page count. If you add a table of contents or abstract then it does contribute to the page count. (Neither a table of contents nor an abstract is necessary.)

Title Page

Title of Special Topic

Lecture Course

Candidate Number: xxxxxx

Introduction

Give a brief introduction to the problem to be solved (you may like to mention the importance of the problem — what Poisson's equation is used for — here).

Explain what method you will use, what other methods are available and why the method you use is a good choice.

Details of the Problem

Note that writing down a weak formulation of a problem is always the first step in a finite element method.

You should describe the weak formulation of the problem, making clear what the definitions are of any notation you use.

It might be interesting to discuss existence and uniqueness of solutions here.

Finite Element Method

Give a brief description of the finite element method for the problem you will solve. State convergence results (no need to prove these as it would be straight from lectures) and be sure to give any restrictions on the problem for these results to hold.

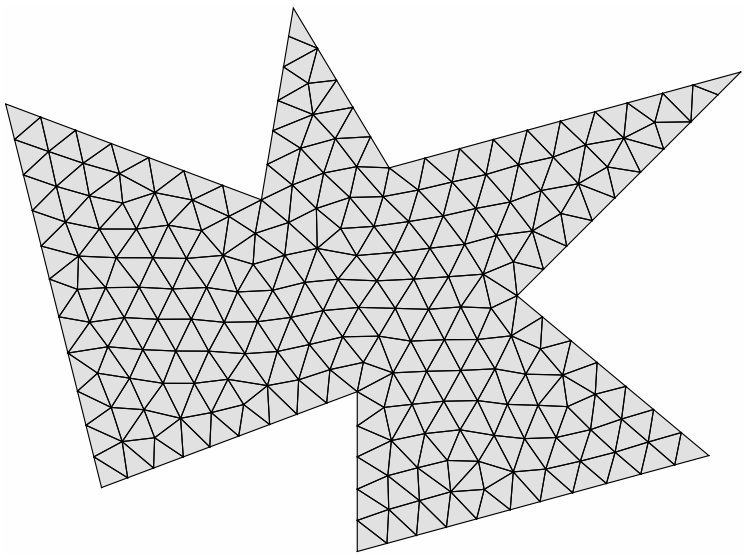
Up to now, nothing we have written will have gone beyond the scope of the lectures. Code was not included in lectures so it might be appropriate to include code snippets, pseudo-code or a short algorithm here to show you really understand how the method works.

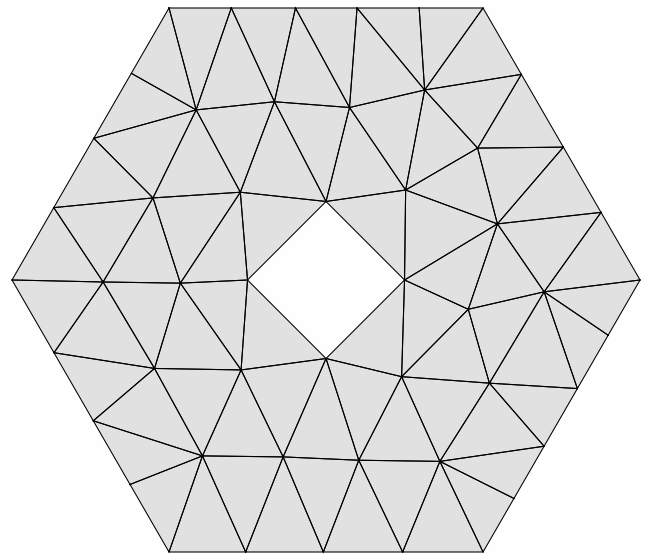
Examples

Now describe what example domains you will use to solve the problem — you have complete flexibility here but be sure to do something interesting, not a rectangle and ideally not a shape made up of rectangles so that the triangulation is more challenging. A domain with a hole could also be fun!

Triangulation

Triangulating a domain that is not rectangular is more challenging as you cannot simply cut the domain into rectangles and then cut those in half diagonally. Fortunately there is plenty of code out there to do this and some of it is written in Matlab. Choose one to use and reference it (webpages should be referenced with the appropriate URL and the date accessed and if there is documentation that should also be referenced).





Quadrature

The finite element method will require you to compute integrals over triangles of the form

$$\int_0^1 \int_0^{1-x} f(x, y) \mathrm{d}y \mathrm{d}x$$

The easiest is to approximate this using a quadrature rule. You should state the rule(s) you will use and the justification for using it (them) as well as giving references.

Numerical Results

First test the code on a problem with a known solution and check:

- does the solution look right?
- do the numerical results agree with the theory (i.e. do the errors converge as expected here)?
- what is the effect of the quadrature rule?

You can present results either as figures or tables or a combination but be sure to discuss the results clearly in the main body of the text.

For a particularly strong project you might like to investigate what happens if the restrictions on the solution for the convergence results to hold, are not valid.

Conclusion

Make sure you write a summary of the project at the end, emphasising any key points in the results.