STRING THEORY I

Lecture 1

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Chapter 1 Introduction

1.1 What is string theory?

(a few words about what string theory is and some motivation)

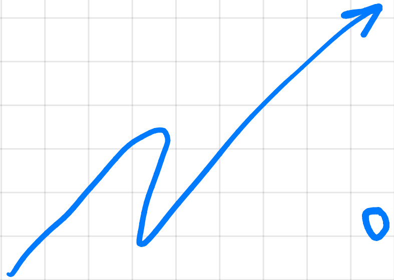
1.2 Historical introduction

1.1 What is string theory?

The starting point of string theory is that it is a theory of fundamental quantum mechanical strings

In QFT: fundamental particles point like objects

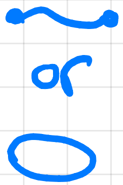
point like particles •

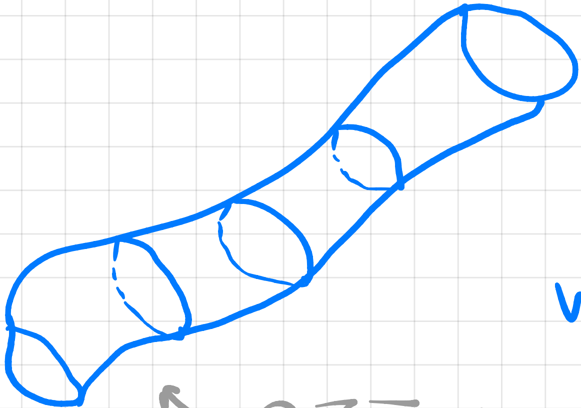


0+1 world line

Instead

strings





1+1 dim world sheet

QFT on world sheet

Perturbative string theory is first quantized S-matrix theory

best developed formulation

2nd quantized theory would be string field theory framework

non perturbative insight post early 1990's

not well understood

May see some in ST II

Key features

- consistently incorporates gravity
  $\Rightarrow$ a theory of quantum gravity
- "unique" theory

- incorporates many other interesting & phenomenologically relevant ingredients from QFT & particle physics
  - non-Abelian gauge symmetries with chiral matter
  - spacetime supersymmetry
  - "unification"

This course:

- bosonic string theory
  - ↳ missing some of the features mentioned above and suffers from serious defects & inconsistencies
  - ↳ however illustrates key ideas & techniques

ST II: learn superstring theory which has been considered as a candidate to some day describe our world.

1.2 Historical motivation

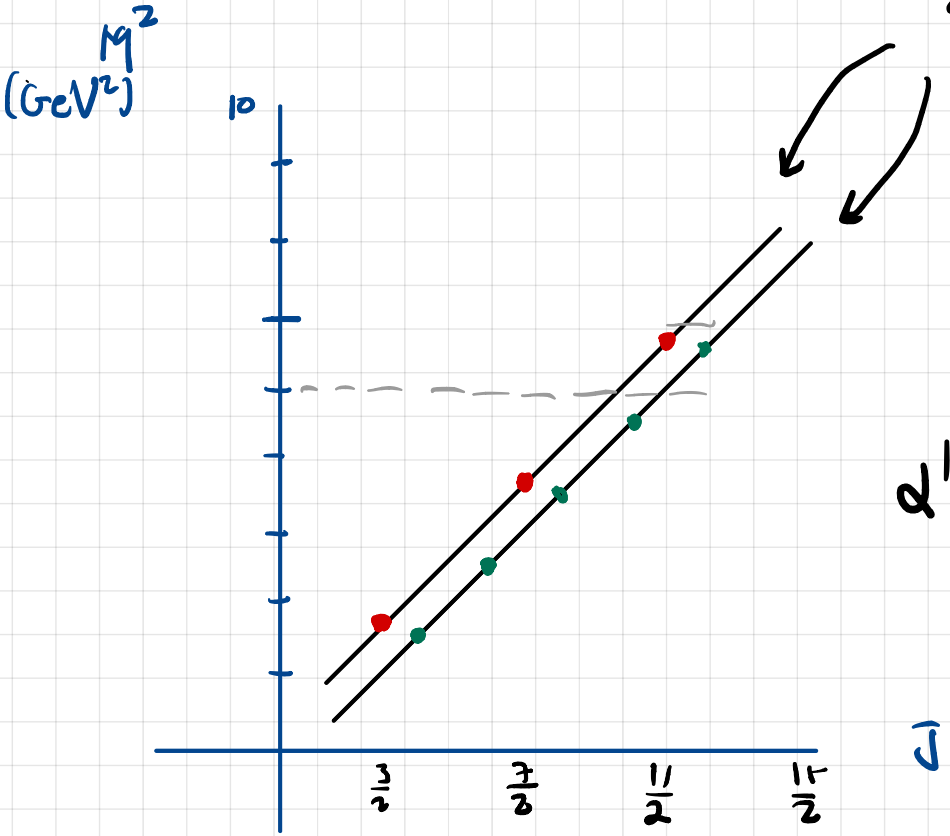
String theory appeared first in the 60's as a theory of strong interactions (the dual resonance models)

50's & 60's: QFT was unsatisfactory as a theory of strong interactions because

① the experimental observation of the large proliferation of the number of hadrons with large masses & spins

② UV (loop) divergences in the computation of perturbative scattering amplitudes particularly for high spin particles

① One of the most important observations was that hadronic resonances appeared in families



Chew-Frautschi plot

Regge trajectories

$$J = \alpha(0) + \alpha' M^2$$

$\alpha(0)$ Regge intercept

$\alpha' \simeq 1\ \mathrm{GeV}^{-2}$ slope

lightest scalar $(J=0)$: $M^2 = -\dfrac{\alpha(0)}{\alpha'}$

fixed $M^2 \to$ max spin $J(M^2)$

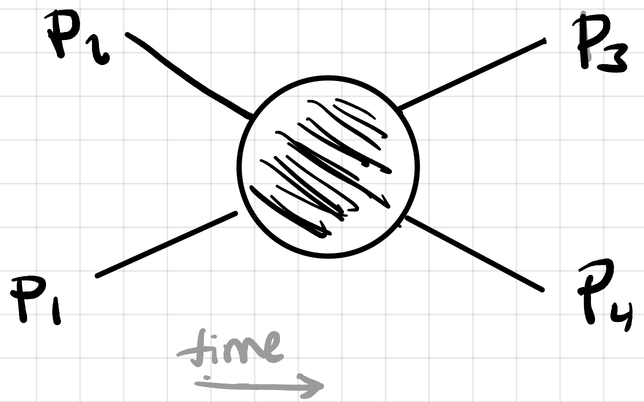
- There where doubts that all these particles where fundamental.

- renormalizable known QFTs: $J = 0, \frac{1}{2}, 1$

Instead people worked within the context of the S-matrix program: construct the S-matrix using a number of general principles like unitarity & analyticity, together with experimental data

② UV difficulties for high spin particles

Consider (for example) the elastic scattering



signature $(-1,+1,+1,\dots)$ so $M^2 = -p^2$

λ_i = flavour quantum numbers $\quad i=1,2,3,4$

Comment: term in scattering amplitude $\propto \mathrm{tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)$

cyclic symmetry $1234 \to 2341$

Mandelstam variables

$$s = -(p_1+p_2)^2 \quad (>0 \text{ for physical elastic scattering})$$
$$t = -(p_1+p_4)^2 \quad (<0 \quad \text{''} \quad)$$
$$u = -(p_1+p_3)^2 \quad (>0 \quad \text{''} \quad)$$

with $s+t+u = \sum m_i^2$

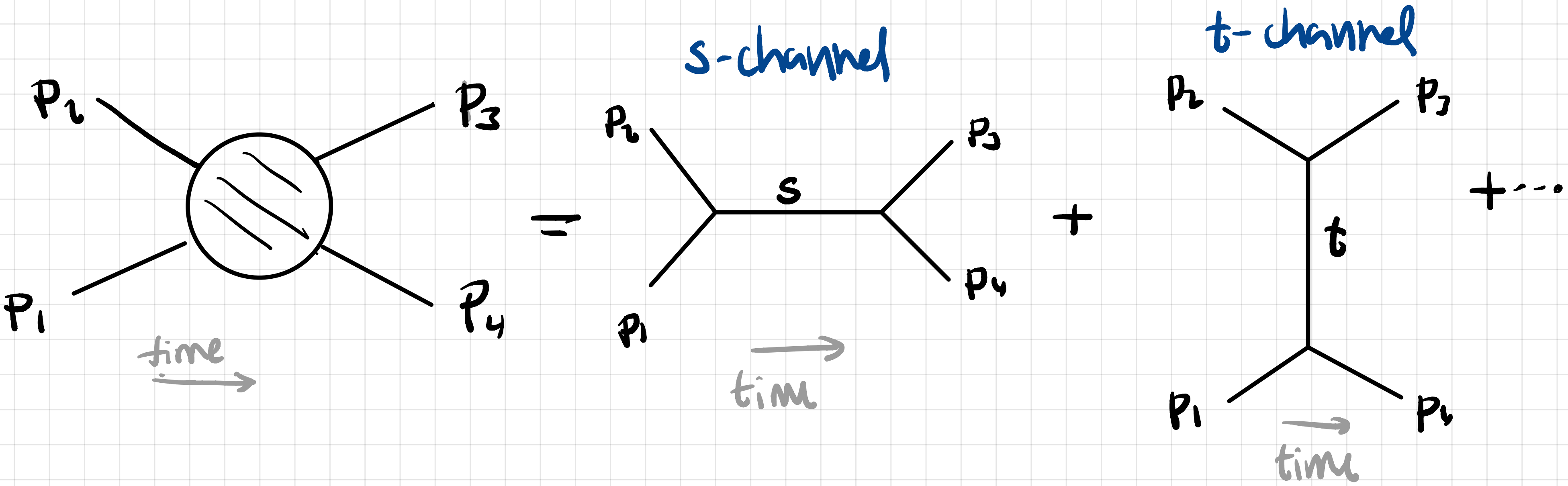
Amplitude $A(s,t)$ depends only on two of Mandelstam variables

Also: as cyclic symmetry of $\mathrm{tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)$

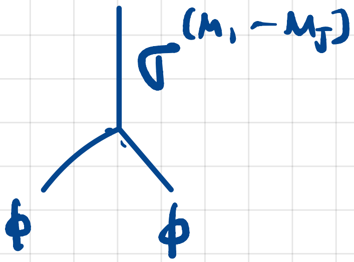
Bose statistics $\Longrightarrow$ $A(s,t)$ must have a cyclic symmetry $p_1 p_2 p_3 p_4 \to p_2 p_3 p_4 p_1$

$\therefore$ $A(s,t)$ invariant under $s \leftrightarrow t$

leading contributions



t-channel exchange of a spin J particle σ



cubic coupling

$$\sim g_J \left(\phi^* \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_J} \phi\right) \sigma^{\mu_1 \dots \mu_J}$$

$$A(s,t) \sim -\frac{g_J^2 (-s)^J}{t - M_J^2}$$

for t fixed
s +ve, large

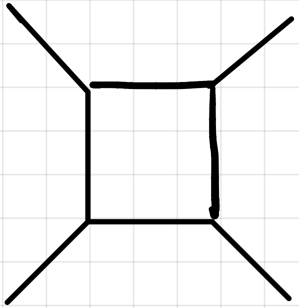
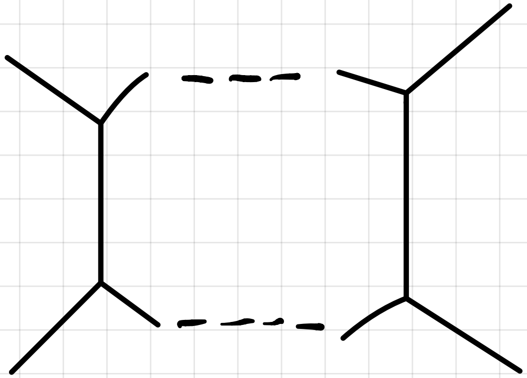
Remarks: • if $J = 0$ cubic coupling $\sim g_0 (\phi^* \phi) \sigma$

$$\Rightarrow \quad A(s,t) \sim -\frac{g_0^2}{t - M_0^2} \longrightarrow 0 \text{ as } t \to \infty$$

• $J > 0$: A is more and more divergent for larger J
(A grows too rapidly at high s)

This UV behaviour is not what was observed in for example pion scattering!

Even worse in loop diagrams:

 $\sim$  $\sim \displaystyle\int \frac{d^D p}{(2\pi)^D} \frac{p^{4J}}{(p^2)^4}$

<u>4 dims:</u>

- $J=0$ safe for scalars
- $J=1$ log divergence potentially renormalizable
- $J>1$ badly divergent ie not renormalizable

If there are particles of various spins exchanged in t-channel

$$A(s,t) \sim -\sum_{J=0}^{J_{max}} \frac{g_J\,(-s)^J}{t-M_J^2} \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\sim} s^{J_{max}}$$

high energy behavior dominated by particle with J_{max}

Bad : • very different from observations

• no s-channel poles

The story might be different if there are infinitely many exchange diagrams:

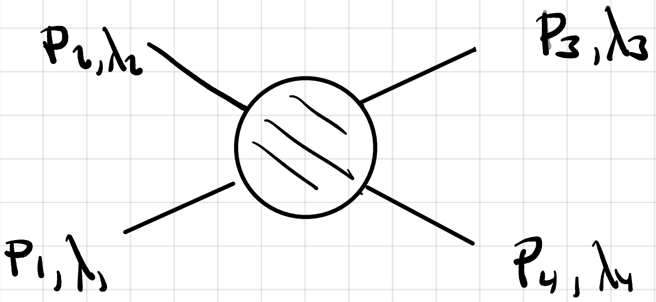
$$A_t(s,t) = -\sum_{J=0}^{\infty} \frac{g_J (-s)^J}{t - M_J^2} \sim \; ?$$

GSW eg e^{-x} smaller for $x \to \infty$ than individual terms in $e^{-x} = \sum_{n=0}^{\infty} \frac{1}{n!} (-x)^n$

As the sum is infinite perhaps s-channel poles arise automatically?

$\boxed{\text{Dolen – Horn – Schmid duality}}$ (1968)

In QFT: need $\boxed{\text{both}}$ s & t channel contributions



$$\sim \; A(s,t)\, \mathrm{tw}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) + \ldots$$

together with $\quad A(s,t) = A_t(s,t) + A_s(s,t)$ both s & t channel contributions

We have $A_t(s,t) = -\sum_J \frac{g_J^2 (-s)^J}{t - M_J^2}$, $A_s(s,t) = \sum_J \frac{g_J^2 (-t)^J}{s - M_J^2}$

due to the $s \leftrightarrow t$ symmetry

Infinite sums: $A_t(s,t)$ might have divergences at some finite values of s
$\Rightarrow$ poles in s-channel

$\Rightarrow$ not obvious that $A_s(s,t)$ needs to be added separately

Instead DHS proposed

Dual model

$$A(s,t) = A_t(s,t) = A_s(s,t)$$

dual description of same physics

In 1968 Veneziano : using the channel duality postulated

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

- $\alpha(s)$ Regge trajectory

  Veneziano postulated $\alpha(s) = \alpha(0) + \alpha' s$

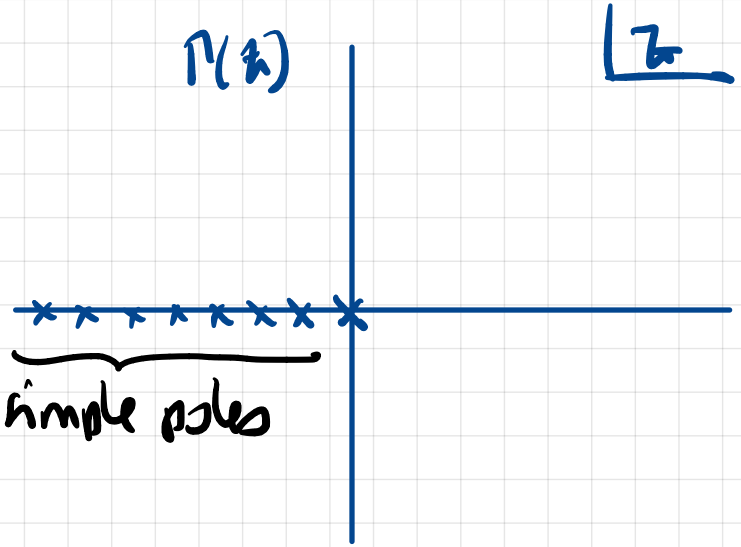
- $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt, \quad (\mathrm{Re}(z) > 0)$ Euler Γ-function

- $B(z, w) = \frac{\Gamma(z)\,\Gamma(w)}{\Gamma(z+w)}$ Euler beta-function

Consider the singularities:

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt, \quad \mathrm{Re}(z) > 0$$



- $\Gamma(z+1) = z\,\Gamma(z)$
- $\Gamma(z)$ has no zeros

Behaviour near singularities: near $z = -n$, n non-negative integer

$$\Gamma(z) = \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n-1)(z+n)} \sim \frac{(-1)^n}{n!}\,\frac{1}{z+n}$$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

$$B(z,w) = \frac{\Gamma(z)\,\Gamma(w)}{\Gamma(z+w)}$$

has simple poles only (at $z=-n$ or $w=-m$ where n, m are +ve integers) so far we have:

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), \alpha(s))$$

with $\alpha(t) = \alpha(0) + \alpha' t$

t-channel poles: $t = \frac{1}{\alpha'}(-\alpha(0)+n)$ $\quad n = 0, 1, 2, \ldots$

s-channel poles: $s = \frac{1}{\alpha'}(-\alpha(0)+n)$ $\quad n = 0, 1, 2, \ldots$

Does $A(s,t)$ satisfy the DHS duality? Yes

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

consider $$B(z,w) = \frac{\Gamma(z)\,\Gamma(w)}{\Gamma(z+w)}$$

Then near a singularity at $w=-n$

$$B(z,w) \sim \frac{1}{z+n}\,\frac{(-1)^n}{n!}\,(w-1)(w-2)\cdots(w-n)$$

Claim: $$B(z,w) = \sum_{n=0}^{\infty} \frac{1}{z+n}\,\frac{(-1)^n}{n!}\,(w-1)(w-2)\cdots(w-n)$$

to sum reproduces all the singularities of B but it precisely B!

[from the fact that $B(z,w) = \int_0^1 dx\, x^{z-1}\,(1-x)^{w-1}$]
(see GSW)

Hence:

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{1}{n!} (\alpha(s)+1)(\alpha(s)+2)\cdots(\alpha(s)+n) \frac{1}{\alpha(t)-n} \qquad (*)$$

& DHS duality means

$$* \quad \underline{A(s,t) = A(t,s)} = -\sum_{n=0}^{\infty} \frac{1}{n!} (\alpha(t)+1)(\alpha(t)+2)\cdots(\alpha(t)+n) \frac{1}{\alpha(s)-n}$$

For the t-channel exchange

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{1}{n!} \underbrace{(\alpha(s)+1)(\alpha(s)+2)\cdots(\alpha(s)+n)} \frac{1}{\alpha(t)-n}$$

$$(\alpha(t) = \alpha(0) + \alpha' t)$$

- singularities are simple poles $\alpha(t)=n$ $\rightsquigarrow$ t-channel exchange of particles with mass $M^2 = \frac{1}{\alpha'}(-\alpha(0)+n)$
- residue at the pole $\alpha(t)=n$: n-th order polynomial in s
  $\rightsquigarrow$ particles of mass $M^2 = \frac{1}{\alpha'}(-\alpha(0)+n)$
  & max spin $J = n$

High energy behaviour of $A(s,t)$: does this solve the UV problem?

$A(s,t)$ in the Regge limit ($s \gg 1$, $t<0$ fixed)

Using Stirling formula $\Gamma(z) \sim \sqrt{2\pi}\, z^{z-1/2}\, e^{-z}$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \sim \Gamma(-\alpha(t))\,(-\alpha(s))^{\alpha(t)} \sim C(t)\, s^{\alpha(t)},$$

$\alpha(t) = \alpha(0) + \alpha' t$

suppressed like $s^{-\alpha' |t|}$

Compare with $A_J(s,t) = \dfrac{g^2(-s)^J}{t-M^2} \sim s^J$, $J = \alpha(t)$ (large s, fixed t)

Hmm: infinite number of particle exchanges in t-channel $\leftrightsquigarrow$ like a single particle with negative spin $J = \alpha(t)$

"Regge-on"

consider now the Veneziano amplitude at high energies $s \gg 1$ for a fixed scattering angle (so t/s fixed).

$$A(s,t) \sim [\underbrace{F(\theta_s)}_{\text{function of the scattering angle } \theta_s}]^{-\alpha(s)}$$

so falls off exponentially fast with s!

✗ Veneziano model

- softer UV behaviour than any QFT
- incorporates particles of high spin without UV divergences

Consider now the Veneziano amplitude at high energies for a fixed scattering angle and $s \gg 1$ with $\frac{t}{s}$ fixed

$$A(s,t) \sim [\underbrace{F(\theta_s)}_{\text{function of the scattering angle } \theta_s}]^{-\alpha(s)}$$

falls off exponentially fast with s !

* Veneziano model has softer UV behaviour than any QFT and incorporated particles of high spin without UV divergences

Virasoro (69), Shapiro (70) model

$$A(s,t,u) = \frac{\Gamma(-\alpha_c(s))\,\Gamma(-\alpha_c(t))\,\Gamma(-\alpha_c(u))}{\Gamma(-\alpha_c(s)-\alpha_c(t))\,\Gamma(-\alpha_c(t)-\alpha_c(u))\,\Gamma(-\alpha_c(u)-\alpha_c(s))}$$

$$\alpha_c(x) = \alpha(0) + \frac{1}{4}\alpha' x \qquad \alpha'(s+t+u) = -16\,\alpha(0)$$

s-channel poles $\quad s = 4(-\alpha(0)+n), \quad n = 0,1,2,\ldots$

t-channel poles $\quad t = \quad$ "

u-channel poles $\quad u = \quad$ "

duality between all 3 channels

max spin @ $\quad m^2 = 4(-\alpha(0)+n) \quad$ is $\quad J = 2n$

Various generalizations

Veneziano model

Virasoro-Shapiro model 1969-1970

⋮

These included

- external particles other than the lightest scalar
- loop amplitudes

⋮

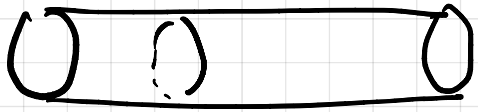
A theory of strings

1970 Nambu + Nielsen + Susskind realized that

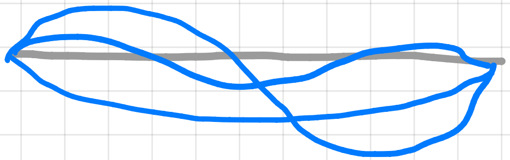
↳ Veneziano & Virasoro + Shapiro models (and their generalizations can be (re) interpreted in terms of a theory where elementary particles are replaced by vibrating relativistic strings

open strings

closed string



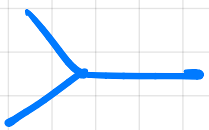
spectrum:



quantized fluctuations of a relativistic string

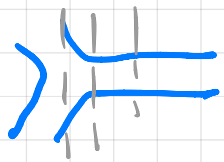
interactions:

For example



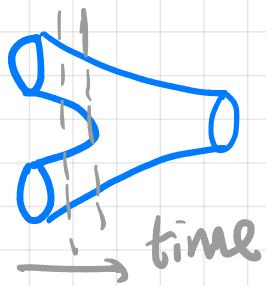
3-point field theoretic particle vertex

$\rightsquigarrow$



3 open-string vertex

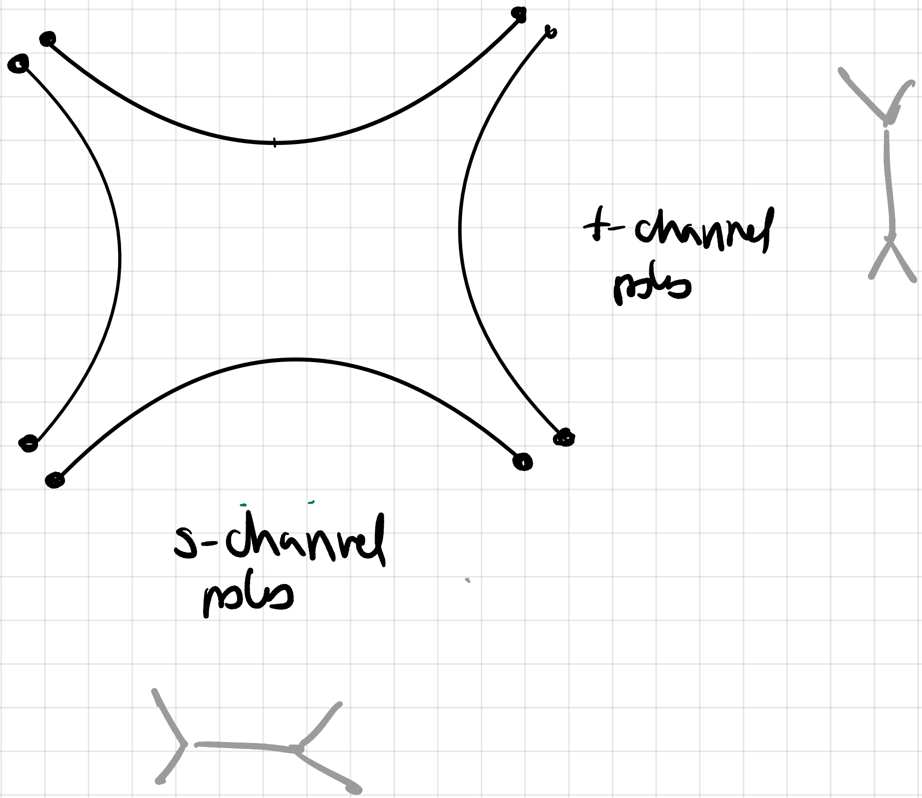
or



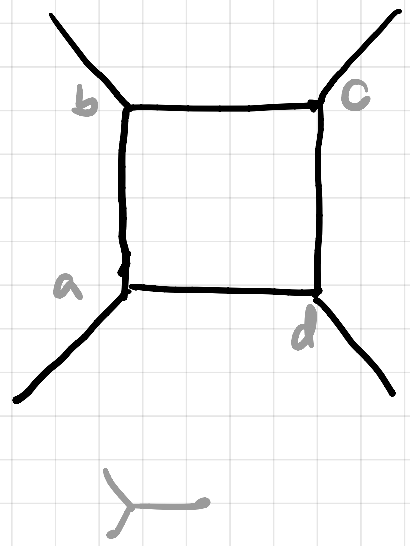
3 closed-string vertex

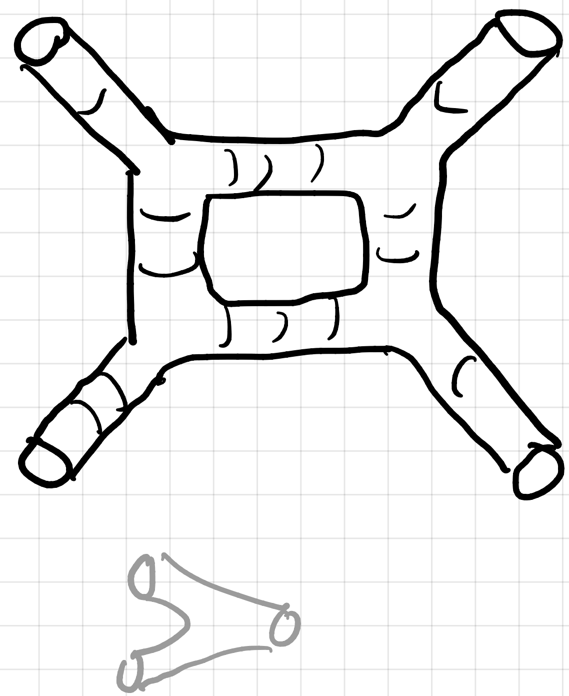
This gives a heuristic justification for the various good properties

duality:

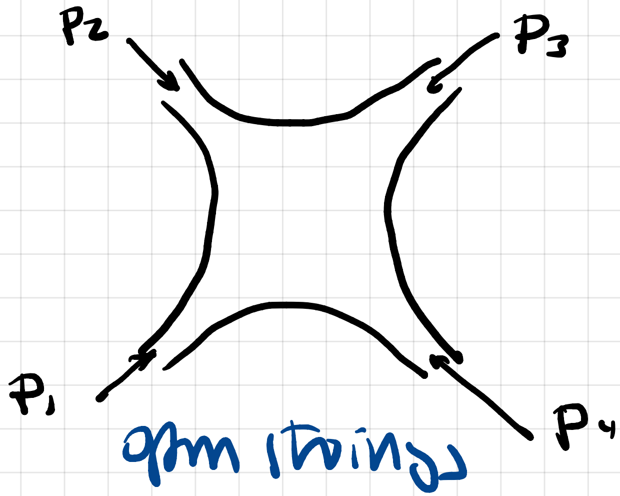


high energy behaviour :

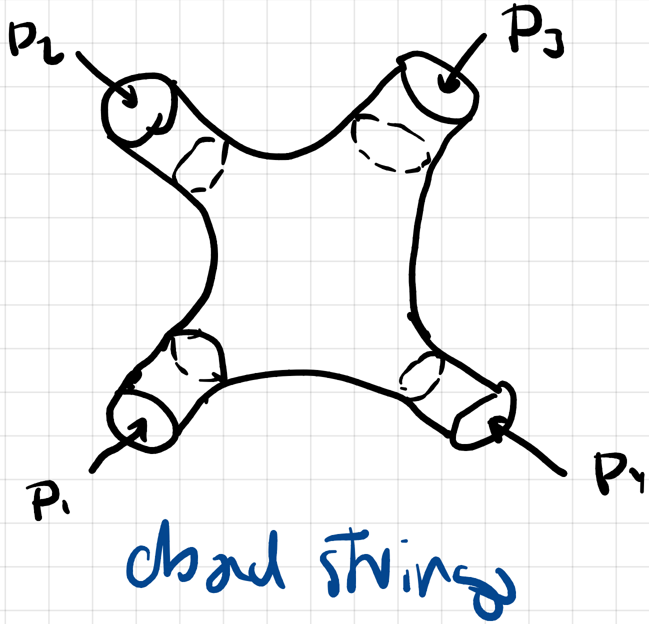




Veneziano $A(s,t)$:



Virasoro-Shapiro $A(s,t,u)$:



Problems

* predicted massless particles
  - spin 1 Veneziano model
  - spin 2 Virasoro-Shapiro (closed strings)

* required more space-time dimensions
  - Veneziano model of bosons $\Rightarrow$ $D = 26$
  - Ramond-Neveau-Schwarz model of bosons & fermions (70) $\Longrightarrow$ $D = 10$

* unitarity of the amplitude not manifest

Dual resonance models abandoned in the 70's in favour of QCD

QCD solves 2,V differently

Gravity and the string scale

Veneziano & Virasoro-Shapiro amplitudes depend on two parameters : $\alpha(0)$ & α' dimensionfull

$[\alpha'] = [M]^{-2}$

original idea in dual resonance models

$\alpha' \sim 1(\text{GeV})^{-2}$ (nuclear physics energy scale)

$-\frac{\alpha(0)}{\alpha'} =$ mass^2 of lightest scalar $= m_\pi^2$

However : the fact that all closed strings contain a massless spin 2 particle suggested the idea that perhaps the theory strings was a theory of gravity as long as

$$\alpha' \sim (10^{19}\ \text{GeV})^{-2}$$

J Scherk & J Schwarz 1974

"Dual models for non-hadrons"

reinterpreted the theory of strings as a unified theory of (quantum) gravity (& other fundamental interactions)

$$S_{\text{zero-slope limit}} = -\int d^4x \sqrt{g} \left(\frac{1}{16\pi G_N} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right)$$

with $\sqrt{8\pi G_N} = g_{str}\sqrt{\alpha'}$

zero slope limit ($\alpha' E^2 \ll 1$) of the VS model gives the tree level Einstein gravity + scalar theory.

<u>Next</u> : Chapter $\boxed{12}$ $\rightarrow$ Classical theory of strings