STRING THEORY T



[Chaster3] Old avaniant quantization

Classical thory: field throwy on the 20in worked-sheet

- $S_{p} = -\frac{T}{a} \int dG dG \left(-\partial_{G} X \cdot \partial_{F} X + \partial_{G} X \cdot \partial_{F} X \right)$
- in the conformal unit gauge Bab=Not.

This is supplemented by the complexity $T_{++} = 0$ & $T_{--} = 0$.

The OCQ approach consists on promoting the canonical fields Xⁿ Q this conjugate momenta $\Pi_T^{nQ} T \partial_T X^n$ to operators and the Poisson bradets {., · IPB

to commutation of apprations

<.,. \₽B ~> i[.,.]

We get the commonical equal time commutation relation

 $\begin{bmatrix} \Pi(\overline{U},\sigma), X^{\nu}(\overline{U},\sigma)\end{bmatrix} = -i 8(\sigma-\sigma') \eta^{\mu\nu}$

(with $[X^{(\sigma)}, X'(\sigma)] = 0$, $[P^{(\sigma)}, P^{(\sigma)},] = 0$)

The operators XM & TTM one humilion

 $\chi^{M} = (\chi^{M})^{\dagger}, \Pi^{M} = (\Pi^{M})^{\dagger}$

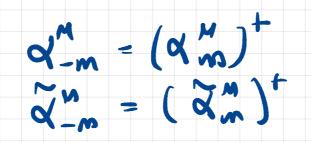
The communition relations for the oscillator modes follow immediately from this:

$$\begin{bmatrix} \hat{p}^{n}, \hat{\chi}^{\nu} \end{bmatrix} = -i \eta^{n\nu} \quad (\text{teisenburg algebra}) \\ \begin{bmatrix} \hat{q}^{n}, \hat{\chi}^{\nu} \end{bmatrix} = m \delta_{m+n, \nu} \eta^{n\nu} \\ \begin{bmatrix} \hat{q}^{n}, \hat{\chi}^{\nu} \end{bmatrix} = m \delta_{m+n, \nu} \eta^{n\nu} \\ \begin{bmatrix} \hat{q}^{n}, \hat{\chi}^{\nu} \end{bmatrix} = m \delta_{m+n, \nu} \eta^{n\nu}$$

The hermiliaty of the operators XM & TT imply that

the two mode operators β^n , \hat{x}^{ν} one Hermilian

while

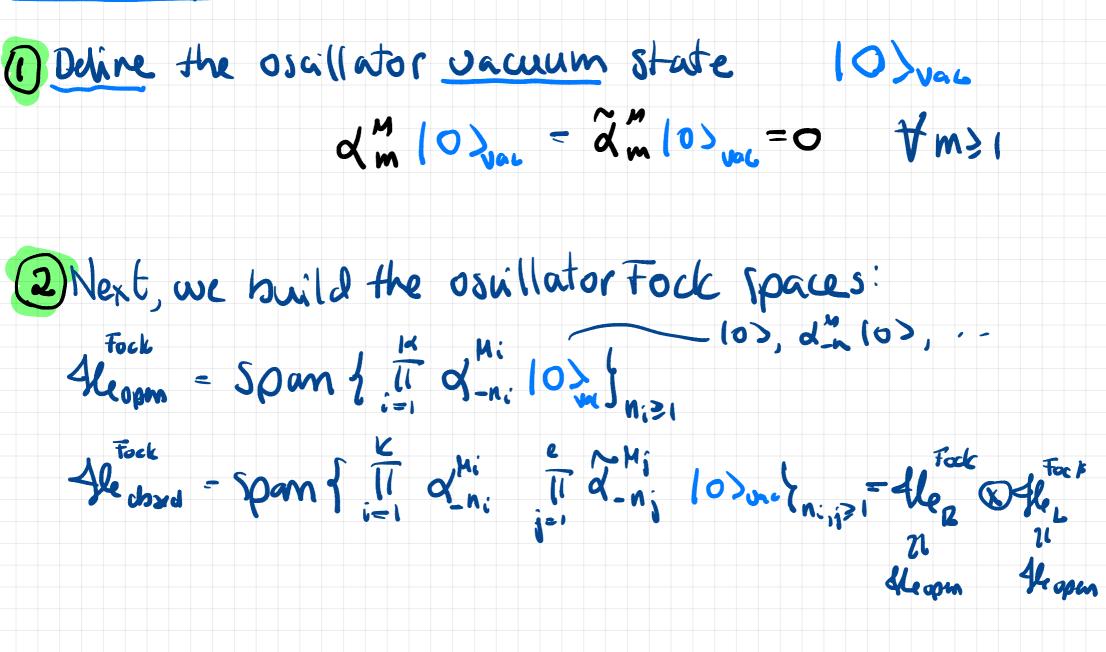


The set $\{a_m^{n}, \widetilde{a}_m^{n}\}$ is an infinite set of havmonic oscillators raining & favering operators.

- We think of
 - W> O W $\alpha_{m}^{M}(\widetilde{\alpha}_{m}^{M})$ annihilation op m 20 60 $q_{-m}^{M}(\widetilde{q}_{-m}^{M})$ creation ops

Now we can compare the Hilbert space

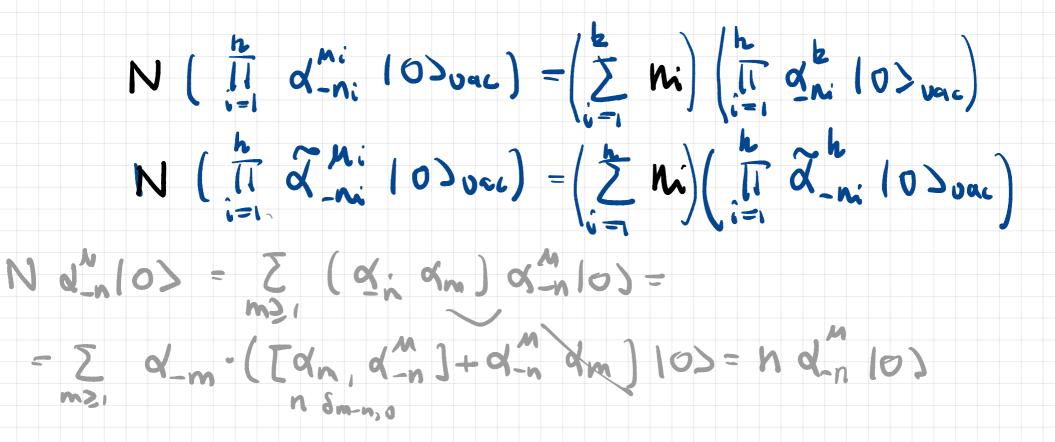
Hilbut space:



Introduce ascillator number oprators

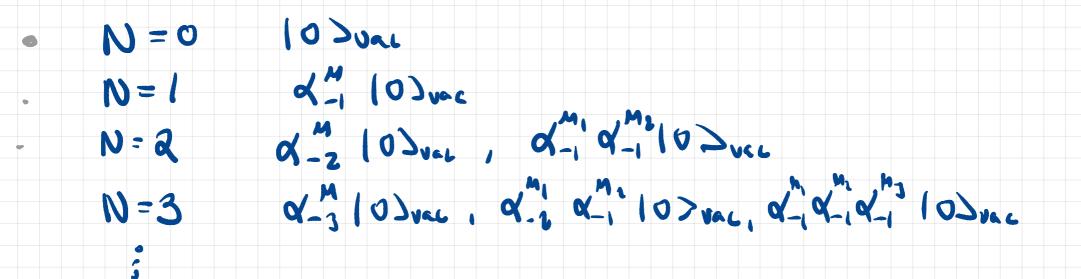


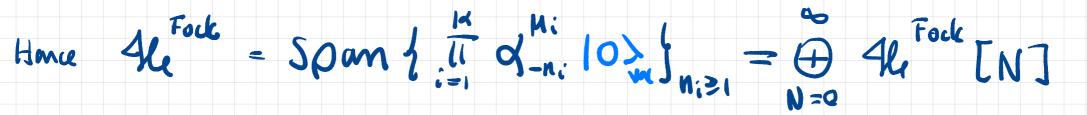
which count oscillators



One can organize the ascillator states into levels, ie states with a given N(B) eignvalue.

For som Trings (or 2-movers, chind (Tring)





(similar proscillators 2)

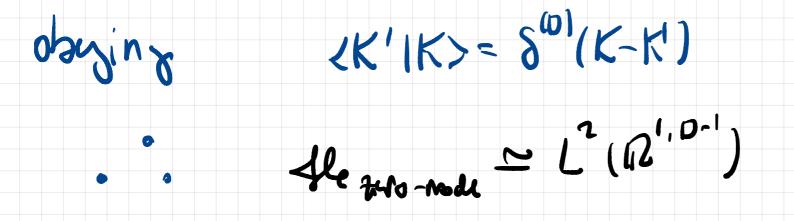


But we have not cet completely sperified the states as we still have the zwo-male.

The ground state is in fact Zis a spatial wave smittin

In momentum space we can door statio to be ciopmistates of the center of mass momentum operator p

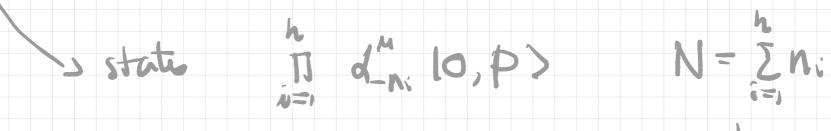
prik) = Krik, Kre R'. 0-1



The Hilbert space is then

Sloopon = (2(R^{1, D-1})) + Fock

Glodovi = (2(R^{1,D-1}) & Gle L & Gle R



momentum

esalue pm Na. Na 10,p> Obzd String

Ploblem (or mt?): Comider the state

1@>= d_, 10; K)

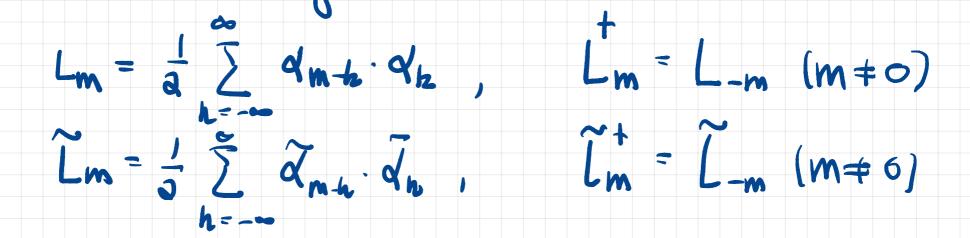


$(q_1 q_2) = 20; K | q_{+1} q_{-1} | 0; K') = n^{\circ} \delta(K - K')$ $[q_{+1}, q_{-1}] = -\delta(K - K')$

Wionz isn! There are negative norm states ghosts! Havern: we have not impossed the constraints

Normal ordering:

recall the Witt-generators

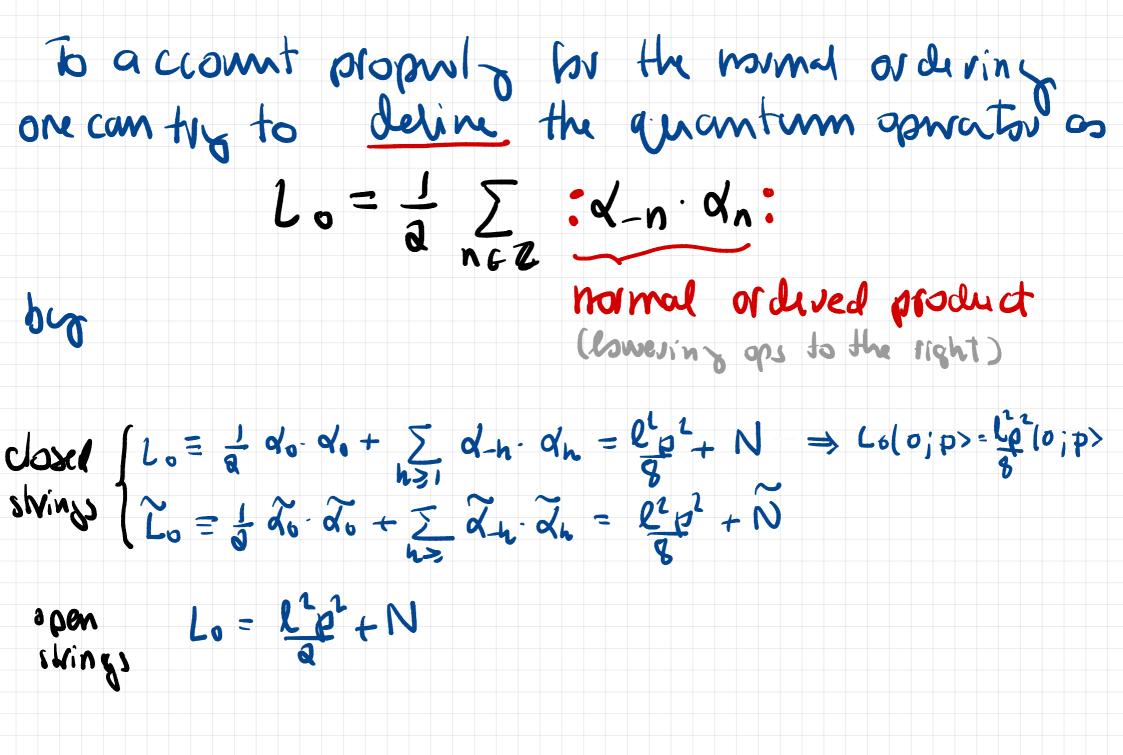


ave quadratic in the oscillators.

The operators dn-n 2 dn commute the unless m=0 to the only "problematic" provator

is Lo (nmilar for Io).

Suppose we chose naively $L_{0} = \frac{1}{4} \sum_{k=0}^{2} d_{k} \cdot d_{k} = \frac{1}{4} d_{0} \cdot d_{0} + \frac{1}{22} d_{-k} \cdot d_{k} + \frac{1}{22} d_{k} \cdot d_{k}$ Thun (oppositions) $L_{o} | 0 ; p \rangle = \left(\frac{l}{2} p^{L} + \frac{i}{2} \sum_{h \ge 1} d_{b} \cdot (d_{-h} | 0, p \rangle) \right)$ $= \left(\frac{l}{2} p^{L} + \frac{i}{2} \sum_{h \ge 1} d_{h} \cdot (d_{-h} | 0, p \rangle) \right) | 0, p \rangle$ $= \left(\frac{l}{2} p^{L} + \frac{i}{2} \sum_{h \ge 1} 2 p \right) | 0, p \rangle$ $= \left(\frac{l^2 p^2}{2} + \frac{l}{1} D \sum_{k \ge l} \frac{l}{2} \right) \left[0, p \right]$ Ecta Similios Régularitation $\sum_{n>1} k = g(-1) = -\frac{1}{12}$ to maybe $= \left(\frac{l}{2}e^{2} - \frac{j}{24}D\right) \left(0, p\right)$



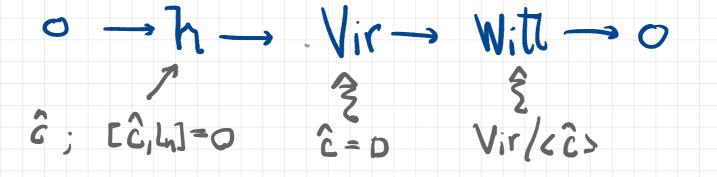
Virasoro algebra We need to check the commutator algebra of the operators Lm (Im)

PS2: a direct comptutation gives

 $[L_{m}, L_{n}] = (m-n) L_{m+n} + \frac{1}{12} (m^{3}-m) S_{m+n}, \sigma$

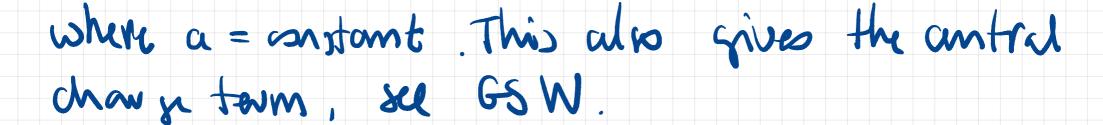
(n'miles for [Gn, Gn] for closed strings) This is the Virasoro algebra with central charge D.

The Viranovo algebra is a central extension of the Witt-algebra



The contral extension is related to an anonabo of the Wey invariance (more (ater).

The global sl(2) dyrbin generated by flo, L, L, f (or { [6, ~, ~] is mt anomabus. Nemark: An alternative way to deal with the normal ordining is to replace the naive opwator to above with



6 - a



Impose the constraints in the quantum theory to identify the physical states

Another proximanisées due to approximance of the Unival change if we define

dephys as the statis 145 which satisfy

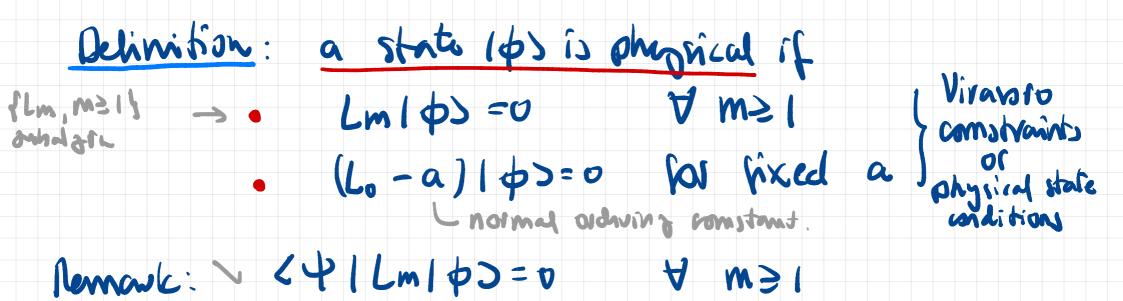
 $Lm | \Psi \rangle = 0$ $\forall m$

In sact, three is a contradiction.



- $\mathbb{E} [l_m, l_n, \mathbb{I} | \Psi \rangle = (l_m, l_n l_n, l_m) (\Psi) = O \quad \forall m, n$ **(**9**)**
- $\frac{[(m, ln](4) = [(m-n)](m+n + \frac{D}{12}(m^{3}-m)](4)}{[(m, l-m)](4) = \frac{D}{12}(m^{3}-m)[(4)]}$ (6)
- Imposing all constraints leads to a trivial Hilbert space when $D \neq 0$!

Instead we define physical states $\phi, \Psi \in \mathcal{A}_{physical}$ by the constraints, $(\Psi \mid L_m \mid \phi) = 0$ $\forall m \neq 0$

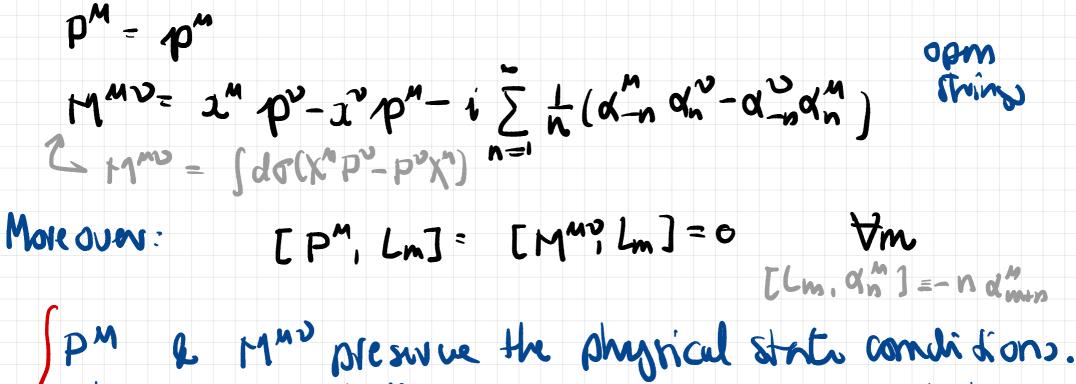


∀m ≤-1

 $\sqrt{241Lm(\phi)} = 2\phi(L-m(\psi)) = 0$

 $\langle \phi | (L_0 - \alpha) | \psi \rangle = 0$

Important remark : the groundars of D-din spare time Poincave grometries have no mormal ordering ombignities as can be seen from



This means that flipm, decomposes into represent ations of TSO(1, D-1) (covariant quantization). Masshell & luul-matching conditions: the Lo (k Lo) condition



mass shell condition

 $(L_0 + \widetilde{L}_0 - a_a) | \psi, K > = 0 \iff (\frac{\ell^2 k^2}{4} + N + \widetilde{N} - 2a) | \psi, K > = 0$

80 $l_{T}^{1}M^{2} = N + \tilde{N} - 2a \implies d'M^{2} = a(N + \tilde{N} - 2a)$ Level matching condition $(L_{0} - \tilde{L}_{0}) | \ell_{0}, K > = 0 \iff (N - \tilde{N}) | \ell_{0}, K > = 0$ so $N = \tilde{N}$

Opm string mass shell condition

 $(L_0-a)|\psi,K\rangle=0 \iff (\frac{l^2p^1}{a}+N-a)|\psi,K\rangle=0 \quad \text{so } a'M^2=N-a$

Level O grand 10, K>

As an exercise, you can show that EN, Ln J = -m Lm

to Lm shifts N-level by -m (similar br NkG) =) at level tero we only need to check the Lo conditions.

The Lo-conditions at level N=0 are

 $\frac{\partial \rho_{M}}{\partial x d} = -4a$

a = 0 massive ground statu

a>o tachyonic ground statu (!)

level one states & dealings with ghosts

Recall : earlier we encountered a problem with regative norm states (genstes) ad level one of the som string (cz: d_, lojK)). We want to see if this issue remains after applying the constraints

Consider a general level one open Tring state $5 \cdot N_{-1} = 15; K > 5 \in \mathbb{R}^{1,D-1}$

polavization vector

and imposse the physical state anditions for Lo & L+1.

The Ln conditions for $n \ge 2$ are satisfied automatically once the $L_0 = L_{+1}$ and imposed.

• mass-shell: - ~ K=~ M2 = 1-a

• L_{+1} condition: $L_{+1} (5 \cdot d_{-1}) |0; K\rangle = M_{NV} 5^{m} [L_{+1}, d_{-1}^{v}] |0; K\rangle$ = $M_{NV} 5^{m} d_{0}^{v} |0; K\rangle = Q(5 \cdot K) |0; K\rangle$

L+115:KS=0 => 5.K=0

OTOH

The norm of a general level 1 state (5, K) is $\langle 5; K|S; K' \rangle = \langle 0; K| (5 \cdot d_+,) (5' \cdot d_-,) |0; K' \rangle$

 $=(S,S')(O;K|O;K')=(S,S')\delta(K-K')$

For S-S': require S'>0 to avoid ghosts

In summon we require : polaritation is light like or space like 520 Kanjuerz plani bation 5.K=0 K is spacelike if a>1 lightlilu if a=1 timelila if a<1 a'K=a-1 K is Timelike, g. K=0 => S spacelike 921 It K is nell, g·K=0⇒5 spacelik or null a = (K is paulile, S·K=0 ⇒ 5 Timilile a>1 => ghosts! Then we reject a>1 ie require a <1 However the around state is a tachyon for ocasi

Critical theory: a=1: K=0 $(5^2 \ge 0)$

5. 2, 10, K) = 15, K) is a massless state

Comidw now the state IK; K> = K·d-10, K) Clearly, this state has zero morm (so it is physical) and has "eon y Endind" polanization

Moreover, this state is orthogonal to all physical Status $K_{1}(1) = (K \cdot 5) \delta(K - K') = 0$ So the longitudinal plas: tation decouples leaving only D-2 physical planitations (like a photon)

However: for a=1 ground state is a tackyon

Note that for the congitudinal degree of freedon the decaroling is due to the fact that IK; KS is a "prve gange" state in the following smr. Consider

 $L_{-10}; K >= e K \cdot d_{-1} lo; K >= e l K; K >$

ie IK; K) created by the action of L-1.

Recalling that L-1 is a generator of a conformal homsformation, we say that IK; K'S is a prove gauge state.

[3] Old covariant quantization

