Chapter 12

Epilogue

In this final chapter, we gesture towards some of the topics that, unfortunately, can't be made to fit into an eight week course but are equally deserving of discussion. I hope you might look into them independently if you have some interest.

Time dependent phenomena and methods

In almost the entirety of this course (with the exception of the discussion of the propagator early on) we have aggressively maintained a focus on aspects of the quantum theory that could be studied through time-independent methods. Of course, the world is dynamical and it is often useful to have more intrinsically time-dependent tools at one's disposal. Some key words in these areas are:

- **Pictures of time evolution.** Using the unitary time evolution operator $U(t_1; t_0)$, one can recast the subject of time evolution as applying to the *operators* of quantum theory rather than the states (this is called the *Heisenberg picture* of time evolution). One can go further and evolve states and operators using different time evolution operators (one involving interactions and one corresponding to free propagation). This leads to the *interaction picture*. This formalism is especially important in perturbative scattering theory.
- **Time-dependent perturbation theory.** A realistic, and therefore important, situation to deal with is when a system is perturbed in a manner that is explicitly time-dependent. This could mean that the "underlying" Hamiltonian is time-dependent (say, because you are on the surface of the Earth which is exposed to electromagnetic radiation from the sun periodically), or that we have an underlying time-independent system which we momentarily disturb in a dynamical way (say, by momentarily hitting a Hydrogen atom with a laser beam). This gives rise to slightly different questions than those we addressed in our analysis of perturbation theory. For example, at what rate will the time-dependent perturbation mediate transitions between some given eigenstates of the original system? If you shine a laser at a gas of Hydrogen atoms, how frequently do you expect to ionise the atoms (knock electrons out of bound states into scattering states)? In the case where the time-dependent effect is small, these problems can be treated by a generalisation of perturbation theory to a time-dependent context.

Remark 12.0.4. **Please observe that aspects of these subjects are present on the course synopsis, but as they did not fall within the material covered in the lectures this year you will not be responsible for them in exams.**

Higher-dimensional scattering

In more than one spatial dimension, the particulars of scattering gets quite a bit more complicated. In particular, the issue of angular dependence takes center stage: given particles incident on a local potential with a fixed momentum, how likely are they to be scattered in any particular direction? This is encoded in something called a differential cross section, and higher-dimensional scattering theory is largely tied up with calculating these cross sections.

The Feynman path integral

An influential "third way" of thinking about quantum theory (in contrast to the algebraic approach of Heisenberg or the differential equation approach of Schrödinger) was supplied by Richard Feynman in a 1948 paper (building on earlier work by himself and others, including Dirac). The idea, roughly, is if we want to compute the propagator,

$$U(x_f, t_f; x_i, t_i) = \langle x_f | U(t_f; t_i) | x_i \rangle \ . \tag{12.1}$$

then by repeated insertions of resolutions of the identity separated by very short time evolution, one arrives at a picture where one should *sum over all possible trajectories* of the particle between the initial and final position. This sum over histories is encoded in the *Feynman path integral*, which is denoted as follows

$$U(x_f, t_f; x_i, t_i) = \int_{x(t_i)=x_i}^{x(t_f)=x_f} [\mathcal{D}x] e^{\frac{i}{\hbar} S[x(t)]} . \tag{12.2}$$

The beautiful result of Feynman's derivation is that the weight with which each trajectory contributes is the (imaginary exponential of) the *classical action* of that trajectory. The integration measure (denoted by $[\mathcal{D}x]$) is a subtle thing to make rigorous sense out of, nevertheless the intuition gained from this formulation has proven invaluable for quantum physicists. Indeed, from this perspective, the WKB approximation that we studied in Chapter 9.4 amounts to performing a stationary-phase approximation for the path integral!

You can learn all about path integrals in, for example, **C7.1 Theoretical Physics**.

Entanglement and quantum information theory

We only touched ever-so-briefly upon the issue of quantum entanglement. A more detailed study of the manipulation of finite quantum systems leads to the subject of quantum computing and quantum information theory, in which entanglement is leveraged to perform computational tasks that would seem impossible using conventional classical methods. You can learn all about this in **C7.4 Introduction to Quantum Information**.