STRING THEORY I

Lecture 16

(final lecture)

7 D-branes

Last lecture: we defined a Dp-brane as a (p+1)-dimensional subspace of target space where the ends of open strings can end
(We refer to this subspace as the Dbrane world volume)

We saw how D-branes appear from T-duality

strings with Neumann boundary conditions $\longleftrightarrow$ Dirichlet boundary conditions

Today: a number of observations about Dbranes
(mostly without proofs $\rightsquigarrow$ just an idea of what these important objects are in the context of string theory)

In this lecture course we studied

- quantised strings (open & closed) in $\mathbb{R}^{1,25}$

  a salient feature is that the massless sector includes

  a graviton (from the closed sector)

  gauge fields (from the open sector)

  Note that we discussed OS with Neumann bcs only

- We also discussed quantised strings in $\mathbb{R}^{1,24} \times S^1_R$

  ↳ new features eg

  CS • states have quantised momentum along S^1_R and a winding quantum number

  • T duality

  OS more complicated; T-duality leads to the notion of D-branes

Last lecture:

OS - T-duality $\longleftrightarrow$

open string with Neumann boundary conditions compactified on S^1_R

dual open string with Dirichlet boundary conditions compactified on $S^1_{\hat{R}}$, $\hat{R} = \alpha'/R$

D25 space-filling brane
↳ open string ends are free to move on spacetime

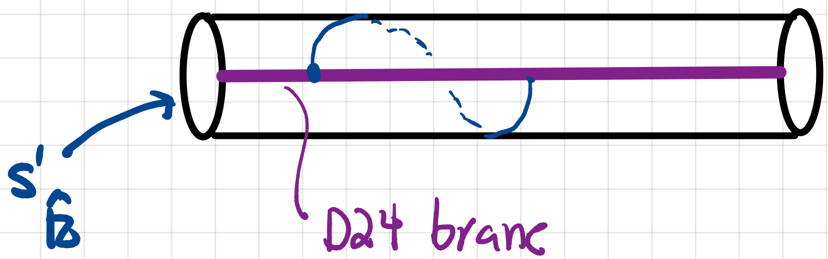
endpoints of the string live on a D24 brane

$p^{25} = \frac{m}{R}$ quantized

no translational symmetry along $S^1_{\hat{R}}$

no winding

string can wind around $S^1_{\hat{R}}$

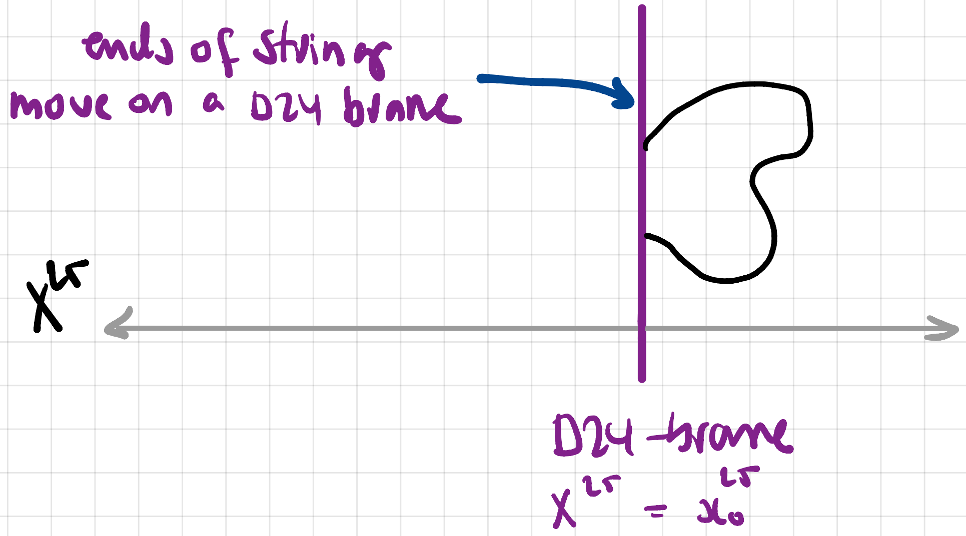


massless sector: (both sides)
25 dimensional U(1) gauge field

7-1 Open strings with Dirichlet b.cs in flat $\mathbb{R}^{1,25}$

(no compactification)

Consider an open string on $\mathbb{R}^{1,25}$ with Dirichlet boundary conditions in one direction (X^{25}) and Neumann boundary in all other directions (X^i $i = 0, \ldots, 24$)



no translational symmetry along X^{25}

space-time symmetry

$$SO(1,25) \longrightarrow SO(1,24)$$

[More generally, one can consider an open string with Dirichlet boundary conditions in $26-(p+1)$ directions and Neumann boundary conditions in $(p+1)$ directions. In this case string ends move on a Dp brane and $SO(1,25) \to SO(1,p) \times SO(25-p)$]

Mode expansion for $X^\mu(\tau, \sigma)$:

Nbc $X^i(\tau,\sigma) = x^i + 2\alpha'\tau p^i + i\sqrt{2\alpha'}\sum_{n\neq 0} \frac{1}{n} \alpha_n^i \cos(n\sigma)$ $i = 0, \ldots, 24$

Dbc $X^{25}(\tau,\sigma) = x_0^{25} + \sqrt{2\alpha'}\sum_{n\neq 0} \frac{1}{n} \alpha_n^{25} e^{-in\tau} \sin(n\sigma)$

$$\boxed{\begin{aligned} X^{25}(\tau,0) &= X^{25}(\tau,\sigma) \\ &= x_0^{25} \end{aligned}}$$

$\boxed{\text{no } \alpha_0^{25} \text{ mode}}$.

$\Rightarrow$ no momentum along X^{25}

(a term $p^a\tau \Rightarrow$ endpoints would not stay at x_0^{25} when $\tau \neq 0$)

quantising the string: mostly as before except

X_0^{25} remains a number

(X_0^{25} is not a parameter, it represents the location of a fixed D brane)

Virasoro operators as before.

Mass-shell condition: $L_0 - 1 = (\alpha' p^2 + N) - 1$

becomes $\alpha' M_{25}^2 = -\alpha' |p|^2 = N - 1$ $\qquad |p|^2 = p \cdot p$ $\leftarrow$ inner product on $\mathbb{R}^{1,24}$

Ground level ($N = 0$): tachyon <u>on the D-brane</u> $\alpha' M_{25}^2 = -1$

Massless spectrum: must be level $N=1$

$$|\xi, \eta; K\rangle = (\xi \cdot \alpha_{-1} + \eta\, \alpha_{-1}^{25})\, |0; K\rangle$$

ground state ($N=0$)

25 dim momentum

(1+24)-dim polarization vector

spacetime scalar

Imposing $L_1 |\xi, \eta; K\rangle = 0$ ($\Rightarrow L_m|\phi\rangle = 0$, $m \geq 2$)

we find that $|\xi, \eta; K\rangle$ is physical if $\xi \cdot K = 0$ with η unconstrained.

$$L_1 |\xi, \eta; K\rangle = \left(\xi_i \left([L_1, \alpha_{-1}^i] + \alpha_{-1}^i L_1 \right) + \eta \left([L_1, \alpha_{-1}^{25}] + \alpha_{-1}^{25} L_1 \right) \right) |0; K\rangle$$

$$= \left(\xi \cdot \alpha_0 + \cancel{\eta\, \alpha_0^{25}} + (\xi \cdot \alpha_{-1} + \eta\, \alpha_{-1}^{25}) L_1 \right) |0; K\rangle$$

$$= (\xi \cdot K + \cancel{(\xi \cdot \alpha_{-1} + \eta\, \alpha_{-1}^{25})(\alpha_1 \cdot \alpha_0)})\, |0; K\rangle = (\xi \cdot K)\, |0; K\rangle$$

null states at level one of the form $L_{-1}|0;K\rangle$:

$$L_{-1}|0;K\rangle = K\cdot\alpha_{-1}|0;K\rangle$$

with $K\cdot K=0$ (from $(L_0-1)(L_{-1}|0;K\rangle)=0$)

Then we have the massless physical states

- 25-dimensional photon $\quad \xi\cdot\alpha_{-1}|0;K\rangle$ U(1) physical state
  $\xi\cdot K=0$

so the Dbrane has a U(1) field on its world volume
(true for any Dp brane)

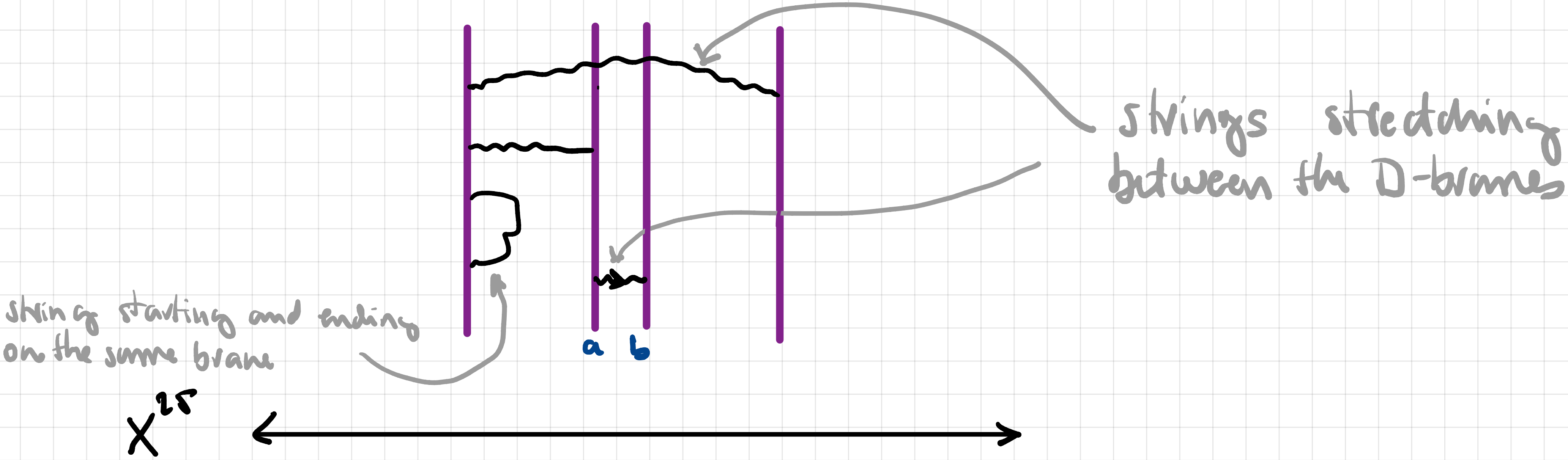
- scalar field $\quad \varphi=\eta\,\alpha_{-1}^{25}|0;K\rangle$

more generally a Dp brane has a massless scalar for each normal direction.

φ can be identified with fluctuations in the position of the D-brane along the transverse X^{25} direction (no proof here!)
see B Zwiebach

7.2 Stretched strings

One can also have systems of D-branes with different classes of open strings (open string sectors)



Consider a string stretched between two parallel D24 branes located at $x_0^{25} = x_a^{25}$ and $x_0 = x_b^{25}$

String endpoints $\quad X^{25}_{ab}(\tau,\sigma=0) = x_a \, , \quad X^{25}_{ab}(\tau,\sigma=\pi) = x_b$

$$X^{25}_{ab} = x_a^{25} + \frac{1}{\pi}\underbrace{\left(x_b^{25} - x_a^{25}\right)}_{\Delta x_{ab}}\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}e^{-in\tau}\sin(n\sigma)$$

$$\alpha_0^{25} = \frac{1}{\pi}\left(x_b^{25} - x_a^{25}\right)$$

mass-shell condition: $\quad M^2_{ab} = -p\cdot p = \left(\frac{x_b^{25} - x_a^{25}}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N-1)$

shift of mass-levels: $\left(\frac{\Delta x}{2\pi\alpha'}\right)^2 = (T\Delta x)^2$ = mass^2 of a classical string stretched between the branes

Spectrum of the stretched string:

• <u>N = 0</u> $\quad |K, ab\rangle$

information about which D brane string ends live
Chan-Paton levels

labels $[a\ b]$

a denotes brane on which end $\sigma = 0$ lives
b denotes brane on which end $\sigma = \pi$ lives

in our case a, b take values 1 or 2; more generally, for N Dbranes they take values 1, …, N

(For a string with ends on the same brane $a = b$)

mass shell condition : $\quad M_{ab}^2 = -\frac{1}{\alpha'} + \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$

tachyon if $\quad |\Delta X_{ab}| < 2\pi\sqrt{\alpha'}$

$\cdot$ **N = 1** $\qquad M^2_{ab} = \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$

$$\left.\begin{array}{l} \zeta\cdot\alpha_{-1}\,|K, ab\rangle \quad (K\cdot\zeta = 0) \\ \eta\,\alpha^{25}_{-1}\,|K, ab\rangle \end{array}\right\}$$

massive vector

on 25 dim space time

$|\eta, \zeta, ab\rangle$

null states

$$L_{-1}|0;K;ab\rangle = K\cdot\alpha_{-1}|K, K;ab\rangle + \frac{\Delta X_{ab}}{\pi}\,\alpha^{25}_{-1}|\eta, K;ab\rangle$$

off

null states

$$L_{-1}|0;K;ab\rangle = \frac{1}{2}\sum_n \alpha_{-1-n}\cdot\alpha_n|\cdots\rangle$$

$$= \frac{1}{2}\left(\alpha_{-1}\cdot\alpha_0 + \sum_{n>0}\left(\cancel{\alpha_{-1-n}\cdot\alpha_n} + \alpha_{-1+n}\cdot\alpha_{-n}\right)\right)|\cdots\rangle$$

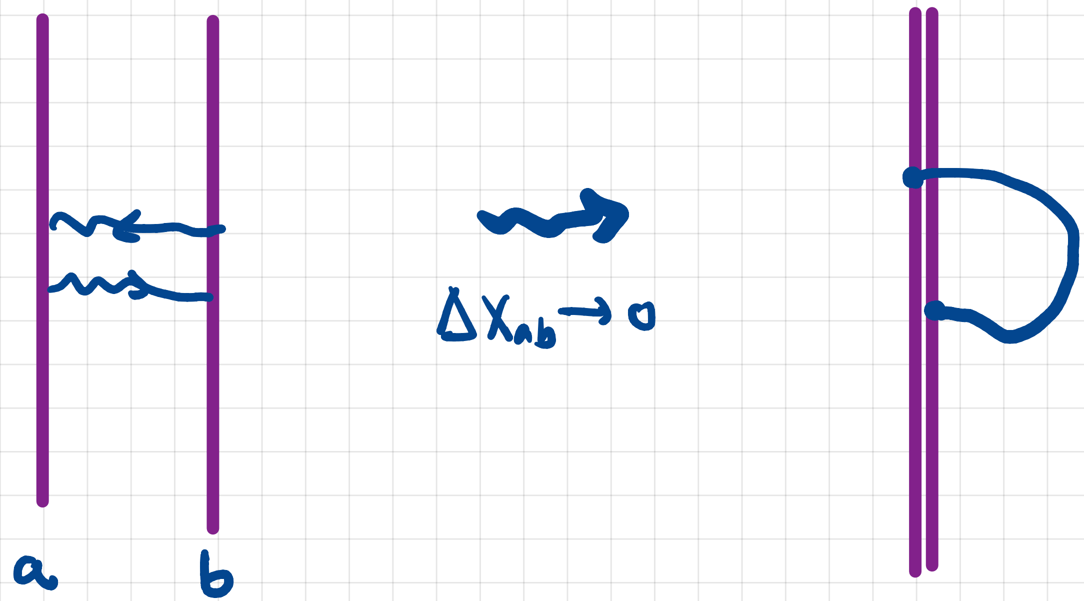
$$= \frac{1}{2}\left[\alpha_{-1}\cdot\alpha_0 + \alpha_{-1}\cdot\alpha_0\right]|\cdots\rangle = (\alpha_{-1}\cdot\alpha_0)|\cdots\rangle$$

$$= \left(\alpha_{-1}\cdot K + \alpha_{-1}^{25}\,\alpha_0^{25}\right)|\cdots\rangle$$

$$= K\cdot\alpha_{-1}|\cdots\rangle + \frac{\Delta X}{\pi}\,\alpha_{-1}^{25}|\cdots\rangle$$

with $K\cdot K = 0$ $\left(\text{from } (L_0-1)(L_{-1}|0;K\rangle) = 0\right)$

Coincident limit: suppose we have N D-branes



stretched string state between D-brane at x_a & D-brane at x_b $\xrightarrow[\Delta X_{ab} \to 0]{}$ massless gauge fields

$$\alpha^i_{-1} |k; ab\rangle + \text{scalars}$$

$a, b = 1, \dots, N$
Chan-Paton labels

N massive vector fields

$$|k^i, ab\rangle$$

N^2 sectors

One can show that the spectrum has a $U(N)$ global symmetry and that these (N^2) states transform in the adjoint representation of $U(N)$

One can choose a basis for these states

$$|p, k; A\rangle = \sum_{a,b} (t^A)^a{}_b \, |p, k; ab\rangle$$

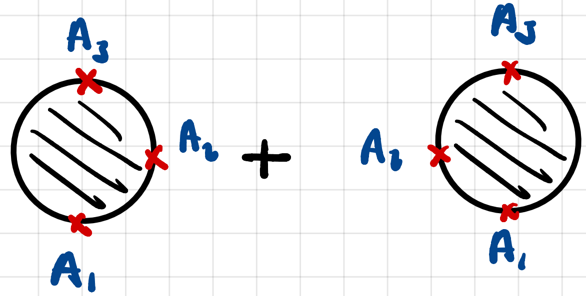
$A = 1, \ldots, N^2$

hermitian basis of $u(N)$

$\mathrm{tr}(t^A t^B) = \delta^{AB}$

Chan-Paton factors

3-point coupling of massless vectors



$$\mathcal{A}(\zeta_1, k_1, A_1;\ \zeta_2, k_2, A_2;\ \zeta_3, k_3, A_3)$$

$$\sim g_0\, \delta(k_1+k_2+k_3) \Big\{ \zeta_1\cdot k_{23}\ \zeta_2\cdot\zeta_3 + \zeta_2\cdot k_{31}\ \zeta_1\cdot\zeta_3 + \zeta_3\cdot k_{12}\ \zeta_1\cdot\zeta_2$$

$$+ \frac{\alpha'}{2}\, \zeta_1\cdot k_{23}\ \zeta_2\cdot k_{31}\ \zeta_3\cdot k_{12} \Big\} \times \mathrm{tr}\left(t^{a_1}[t^{a_2}, t^{a_3}]\right)$$

gives the 3-point vertex for the $U(N)$ non-Abelian gauge theory

$$\mathcal{L} = \underbrace{-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang-Mills}} - \frac{2i}{3}\alpha'\,\mathrm{Tr}(F_\mu{}^\nu F_\nu{}^\omega F_\omega{}^\mu) + \text{scalars}$$

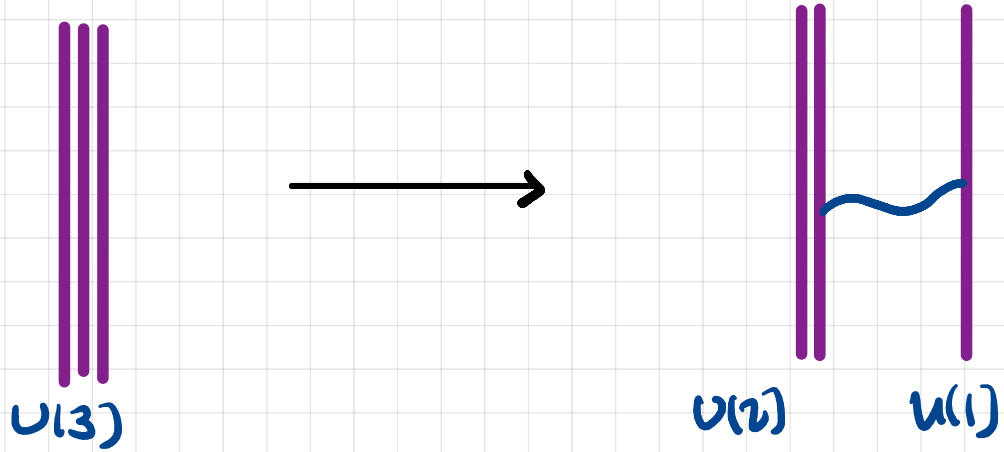
EFT action on D24 brane

α' corrections

One can also obtain this from the β-function

(needs boundary couplings and boundary renormalization flow)

D-brane picture of the Higgs mechanism



stack of 3 Dbranes

$$M_{ab}^2 = \left(\frac{|\Delta x|}{2\pi\alpha'}\right)^2$$

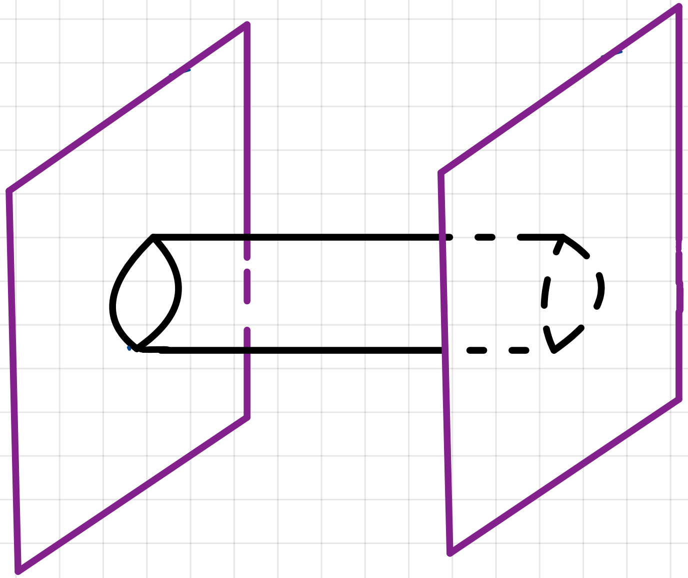
mass of string stretched between branes

D-branes as dynamical objects?

If they are, maybe they need to be included in the perturbative description of strings? how?

Estimate mass scale relevant to D-branes by computing its tension

closed string exchange between two Dbranes



gravitational coupling $\kappa \sim g_0^2$

$$A \sim \kappa^2 T_p^2 \sim (g_0)^0$$

D-brane tension

genus 0

$$T_p \sim \frac{1}{g_0^2} \Rightarrow$$ D-branes are massive "non perturbative" objects

What happens then as $g_0 \to \infty$?

Polchinski 1995, "duality revolution"

Final remarks:

We have seen that the theory of quantised string has a very rich structure

- quantised gravity (at low energies we obtain Einstein's gravity)
- gauge fields
- consistency of the theory $\rightsquigarrow$ fixes dimension of spacetime
- S-matrix with good UV behaviour

More

- dualities
- CFT
- emergence of non-perturbative Branes

Improvements:

- remove tachyons $\rightsquigarrow$ (ST2) superstrings
  (fermions in 2dim NLσM : supersymmetric WS theory)
- spacetime fermions $\rightsquigarrow$ ST2
- superstring theory (ST2) $\rightsquigarrow$ spacetime dim $= 10$

  ⋮

- strong coupling
  Black hole physics
  realistic phenomenology
  mathematical structure

End of String Theory I

Thanks!