

## Exercises for Practical #1

In this practical you will learn to construct matrices, solve linear systems, and the basics of writing MATLAB code in a `.m` file script.

1. Create a file called `fin_diff.m` in your `matlab_folder` and add suitable comments at the top. Use it to write down your solutions to the following problems.
2. Check the help for `diag` and use it to construct the  $16 \times 16$  matrix, with  $h = 2/16$ :

$$D2 = \frac{1}{h^2} \begin{pmatrix} -2 & 1 & 0 & 0 & \dots & 0 & 1 \\ 1 & -2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & \dots & 0 & 1 & -2 \end{pmatrix}$$

We will be using this matrix to solve ODEs on the interval  $[-1, 1]$ . In general, for a matrix of size  $N \times N$ , we have  $h = 2/N$ .

3. Read about `toeplitz` and use it to construct `D2` again for `N = 16`.
4. Adapt the code from questions 2 and 3 to allow for an  $N \times N$  matrix (replacing also  $h$  by  $2/N$ ), where `N` is a variable which can be easily adjusted.
5. Define two vectors
 

```
x = -1:2/N:1-2/N;
f = sin(pi*x');
and plot D2*f against x.
```
6. What does the above graph show? (Hint: google “finite difference”.) Amend your comments at the start of the file `fin_diff.m` accordingly. Plot (on a `subplot`) the error in the `D2*f` approximation.
7. Let  $f(x) = e^{\sin \pi x}$ . Using the same `x` as in problem 5 with  $N = 256$ , compute `D2*f` and plot it on top of a plot of  $f(x)$ . Now compute the second derivative of  $f$  by hand! Define an anonymous function:
 

```
fpp = @(x) [expression for 2nd derivative].
```

 Plot `fpp(x)` on top of the existing plots.  
 Now plot `D2*f - fpp(x)` in a new figure.

8. With `D2`, `x` and `f` as in **problem 5**, define
- $$\mathbf{A} = (\mathbf{D2} - \text{eye}(N))/(1+\pi^2)$$
- and plot `A\f` against `x`.  
(Congratulations!! You have just solved your first differential equation in MATLAB! - do you know what it was?)
9. (Advanced) Repeat all of the above using *sparse* matrices. (Hint: check out the `spdiags` command.)
10. (Advanced) What is the closed-form expression for the eigenvalues of the matrix `D2`? Compare with a numerical approach in MATLAB. (Hint: check out the `eig` and `eigs` commands.)