Math C5.4, Networks, University of Oxford
Problem Sheet 4 (Week 5)

Renaud Lambiotte, Mathematical Institute

1. *Reading.* Browse through Sections VI and VII of the lecture notes.
2. *Graph partitioning.* Ex.VI.1: Implement numerically a spectral bipartitioning method, taking as an input a graph, and the desired size of the clusters.
3. *Modularity.*
   (a) Apply modularity optimization techniques implemented in the library of your choice on some examples and visualise the results.
   (b) Ex.VI.2 : Write a function that takes a graph and its partition as an input and returns its modularity. Verify the values obtained in the previous exercise.
   (b) Ex.VI.4 : In the Louvain method, the efficiency of the algorithm partly resides in the fact that the variation of modularity Δ_{ij} obtained by moving a vertex i from its community to the community of one of its neighbors j can be calculated with only local information. In practice, the variation of modularity is calculated by removing i from its community $\Delta_{remove;i}$ (this is only done once) then inserting it into the community of j $\Delta_{insert;ij}$ for each neighbor j of i. The variation is therefore: $\Delta_{ij} = \Delta_{remove;i} + \Delta_{insert;ij}$. Derive analytically $\Delta_{remove;i}$ when removing node i from its community C_i.
4. *Critically reading journal articles.* Read Santo Fortunato, Marc Barthelemy, Proc. Natl. Acad. Sci. USA 104 (1), 36-41 (2007) and write a 1-page summary, typeset using LaTeX, of the main findings and methods of this article, as well as its strength and limitations.