Algorithm 1 Pseudo-code of the community detection algorithm.

```
Community detectionG initial graph
repeat
    Place each vertex of G into a single community
    Save the modularity of this decomposition
    while there are moved vertices do
        for all vertex n of G do
            c ← neighboring community maximizing the modularity increase
            if c results in a strictly positive increase then
                move n from its community to c
            end if
        end for
    end while
    if the modularity reached is higher than the initial modularity, then
        end ← false
        Display the partition found
        Transform G into the graph between communities
    else
        end ← true
    end if
until end
```

3.4 Modularity increase

The efficiency of the algorithm partly resides in the fact that the variation of modularity Δ_{ij} obtained by moving a vertex i from its community to the community of one of its neighbors j can be calculated with only local information. In practice, the variation of modularity is calculated by removing i from its community $\Delta_{remove;i}$ (this is only done once) then inserting it into the community of j $\Delta_{insert;ij}$ for each neighbor j of i. The variation is therefore: $\Delta_{ij} = \Delta_{remove;i} + \Delta_{insert;ij}$.

3.4.1 Remove a vertex from its community

Let us calculate the variation of modularity when a vertex x is removed from its community. Assume that x is not alone in its community (the opposite case is trivial). By removing x from its community, the size of the community of x is decreased $C_x \to C_x \backslash \{x\}$ and a new community only containing x is created $C_x^{'}$. The original modularity is:

$$\begin{aligned} Q = & \sum_{C} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \\ = & \sum_{C \neq C_x} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] + \frac{1}{2m} \sum_{i,j \in C_x} \left[A_{ij} - \frac{k_i k_j}{2m} \right] , \end{aligned} \tag{7}$$

and after removing the vertex x from C_x, the modularity becomes:

$$\begin{aligned} Q^{'} = & \sum_{C \neq C_x} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] + \frac{1}{2m} \sum_{i,j \in C_x \setminus \{x\}} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \\ & + \frac{1}{2m} \left[A_{xx} - \frac{k_x^2}{2m} \right] \\ = & \; Q - \frac{1}{m} \sum_{i \in C_x \setminus \{x\}} \left[A_{ix} - \frac{k_i k_x}{2m} \right] , \end{aligned} \tag{8}$$

where we used the fact that A_{ij} is symmetric. The modularity variation is given by:

$$\Delta_{remove} = Q^{'} - Q = -\frac{1}{m} \sum_{i \in C_x \setminus \{x\}} \left[A_{ix} - \frac{k_i k_x}{2m} \right] . \tag{9}$$

3.4.2 Inserting a vertex into a community

Let us consider the situation where a vertex x is alone in a community and where it is moved into another community C_1. The original modularity is:

$$\begin{aligned} Q = & \sum_{C} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \\ = & \sum_{C \neq (C_x, C_1)} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] + \frac{1}{2m} \sum_{i,j \in C_1} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \\ & + \frac{1}{2m} \left[A_{xx} - \frac{k_x^2}{2m} \right] , \end{aligned} \tag{10}$$

and after movement of x to C_1, which becomes $C_1^{'}$, the modularity becomes:

$$\begin{aligned} Q^{'} = & \sum_{C \neq C_1^{'}} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] + \frac{1}{2m} \sum_{i,j \in C_1^{'}} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \\ = & \sum_{C \neq C_1^{'}} \frac{1}{2m} \sum_{i,j \in C} \left[A_{ij} - \frac{k_i k_j}{2m} \right] + \frac{1}{2m} \sum_{i,j \in C_1} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \\ & + \frac{1}{m} \sum_{i \in C_1} \left[A_{ix} - \frac{k_i k_x}{2m} \right] + \frac{1}{2m} \left[A_{xx} - \frac{k_x^2}{2m} \right] . \end{aligned} \tag{11}$$

The modularity variation is given by:

$$\Delta_{insert} = Q^{'} - Q = \frac{1}{m} \sum_{i \in C_1} \left[A_{ix} - \frac{k_i k_x}{2m} \right]. \tag{12}$$

In both cases, whether it concerns removal or insertion, the calculations of variations are performed using only local information on x and its neighbors.