Exercise sheet 4*

(for the last class, at the beginning of Trinity Term)

Exercise 1.1. *Prove Lemma 3.7 in the notes.*

Exercise 1.2. *Let $f : X \to S$ be a morphism of schemes with the property (P), where (P) is one the following:*

- *affine;*
- *an open immersion;*
- *a closed immersion;*
- *locally of finite type;*
- *quasi-compact.*

Let $g : S' \to S$ be a morphism of schemes and let $f' : X_{S'} = X \times_S S' \to S'$ be the morphism obtained from f by base-change. Show that f' also has property (P). We say that property (P) is invariant under base-change.

Exercise 1.3. *Let $\phi : A \to B$ be a morphism of integral rings. Suppose that $\mathrm{Spec}(\phi) : \mathrm{Spec}(B) \to \mathrm{Spec}(A)$ has dense image. Show that ϕ is injective.*

Exercise 1.4. *Let X be a noetherian scheme and let L, M be line bundles on X.*

• *Suppose that L is ample. Show that for sufficiently large $n \geqslant 0$, the line bundle $L^{\otimes n} \otimes M$ is ample.*

• *Suppose that L and M are ample. Show that the line bundle $L \otimes M$ is ample.*

Exercise 1.5 (optional)**.** *Let X be a reduced noetherian scheme. Suppose that each irreducible component of X, viewed as closed reduced subscheme of X, is affine. Show that X is affine.*

*For the course C2.6 Introduction to Schemes, Oxford, Hilary Term 2018