String Theory I: Problem Sheet 4

Hilary Term 2018

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1. Low energy effective theory in spacetime

The conditions for Weyl invariance of the bosonic string sigma model at leading order in the α' expansion take the form $\beta^G_{\mu\nu} = \beta^B_{\mu\nu} = \beta^\Phi = 0$ with

$$\begin{aligned}
\beta^G_{\mu\nu} &= R_{\mu\nu} + 2D_\mu D_\nu \Phi - \frac{1}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} \ , \\
\beta^B_{\mu\nu} &= -\frac{1}{2} D^\lambda H_{\lambda\mu\nu} + D^\lambda \Phi H_{\lambda\mu\nu} \ , \\
\beta^\Phi &= \frac{D-26}{6} + \alpha' \left(-\frac{1}{2} D^2 \Phi + D_\lambda \Phi D^\lambda \Phi - \frac{1}{24} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) \ .
\end{aligned}$$

In this problem you will check a number of good features of these equations, which we interpret as equations of motion for the spacetime fields G, B, and Φ.

1. Show that these Weyl invariance conditions are equivalent to the Euler-Lagrange equations for the spacetime effective action

$$S_{26} = \frac{1}{2\kappa_0^2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[-\frac{2(D-26)}{3\alpha'} + R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + 4\partial_\mu \Phi \partial^\mu \Phi + O(\alpha') \right] \ .$$

2. The conditions, $\beta^G_{\mu\nu} = \beta^B_{\mu\nu} = 0$ imply (one-loop) conformal invariance of the sigma model on a flat worldsheet. Show that for a solution of these equations, one has automatically that β^Φ is constant.

   This is an important consistency check since in this case, β^Φ should be identified with the central charge of the conformal field theory.

3. Consider the simple "linear dilaton" background,

$$G_{\mu\nu}(X) = \eta_{\mu\nu} \ , \qquad B_{\mu\nu}(X) = 0 \ , \qquad \Phi(X) = V_\mu X^\mu \ .$$

   This background can solve the Weyl invariance conditions with $D < 26$, even though on a flat worldsheet it is identical to the usual free string action.

   Derive this as an exact statement (as opposed to at leading order in α') by computing the stress tensor and Virasoro central charge for the (free!) linear dilaton theory.

4. What is the on-shell three-point dilaton vertex implied by the spacetime action S_{26}? Verify that the result agrees with the worldsheet worldsheet computation.

   [*This is a fairly annoying computation but the result should be very simple in the end.*]

2. Circle compactification of the bosonic string

In lecture we (will have) considered compactification of the 26-dimensional closed bosonic string on a spacetime circle of radius R: $X^{25} \sim X^{25} + 2\pi R$. In this problem you will work out some details and extensions of that story.

1. At generic compactification radius, the massless spectrum of physical string states matches our expectation from dimensional reduction of the particle spectrum of the low energy effective theory.

   Perform the dimensional reduction of the *action* S_{26} to 25 dimensions (*i.e.*, take all 26-dimensional fields to be independent of the X^{25} coordinate and rewrite the action in terms of 25-dimensional fields).

2. Show that for the special choice $R^2 = 1/2$ in string units (*i.e.*, $R = \sqrt{\alpha'}$) there are additional massless states in the physical string spectrum. What are their space-time quantum numbers? What is your interpretation of these additional massless states?

Send corrections, especially about minus signs, to `christopher.beem@maths.ox.ac.uk`.