

Constructive Mathematics. QS 1

(M) \Rightarrow It is suggested that you use MATLAB.

1. (a) Use Euclid's method to calculate $hcf(19397, 1904)$. Show all of the steps.
(b) (M) Use Euclid's method to compute $hcf(9506112, 4183179)$.

2. (a) Find all solutions $x, y \in \mathbb{Z}$ of $163x + 16y = 1$.
(b) Find $hcf(2023, 812)$ and all solutions of

$$2023x + 812y = 28, \quad x, y \in \mathbb{Z}.$$

3. Suppose that $a, b \in \mathbb{N}$ and $c \in \mathbb{N}$ divides ab . Prove that if $hcf(a, c) = 1$, then c divides b .

4. Prove that if $a, b, c \in \mathbb{N}$ with $hcf(a, c) = 1 = hcf(b, c)$, then $hcf(ab, c) = 1$.

5. Perform the division algorithm for the real polynomials $x^3 + 2x^2 - 5x - 6$ and $x^2 + 3x - 10$. What is the remainder? By Euclid's algorithm, find all roots common to these two polynomials.

6. The Legendre polynomials (which you may come across in various contexts later) are defined by $P_0(x) = 1$, $P_1(x) = x$ and for $k \geq 1$ by the 3-term recurrence relation

$$P_{k+1}(x) = \frac{2k+1}{k+1} x P_k(x) - \frac{k}{k+1} P_{k-1}(x).$$

Prove that for no $k \in \mathbb{N}$ can P_k and P_{k+1} have any common roots.