**Part A Quantum Theory, 2014**
**Problem Sheet 2 (of 4)**

1. Consider a particle of mass $m$ confined to a box in three dimensions, with potential

$$V(x,y,z) \quad = \quad \begin{cases} 0\,, & 0<x<a\,,\ 0<y<b\,,\ 0<z<c\,, \\ +\infty\,, & \text{otherwise}\,, \end{cases}$$

where $(x,y,z)$ are Cartesian coordinates. By separating variables in the stationary state Schrödinger equation, show that the allowed energies of the particle are

$$E_{n_1,n_2,n_3} \quad = \quad \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}+\frac{n_3^2}{c^2}\right)\,,$$

where $n_1, n_2, n_3$ are positive integers, and find the corresponding normalized wave functions.

2. Consider the wave-function:

$$\psi_1(x) = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} 2\xi\, \mathrm{e}^{-\xi^2/2}\,,$$

where $\xi = \sqrt{\frac{m\omega}{\hbar}}x$. *(This is the first excited state wave function for a harmonic oscillator of frequency $\omega$, but you don't need to know that to do the question.)*

Show that $\psi_1(x)$ is normalized. Compute the expected values of $x$ and $|x|$ in the state $\psi_1$.

3. A particle of mass $m$ and charge $q$ moves on the $x$-axis under the influence of a harmonic oscillator potential of angular frequency $\omega$, and a constant electric field $\mathcal{E}$. The potential is

$$V(x) = \frac{1}{2}m\omega^2x^2 - q\mathcal{E}x\,.$$

Show that the energy levels are

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{q^2\mathcal{E}^2}{2m\omega^2}\,,$$

where $n$ is a non-negative integer. [*Hint: change the space variable.*]

4. A particle of mass $m$ moves in three dimensions under the influence of the potential

$$V(x,y,z) = \frac{1}{2}m\omega^2(x^2+y^2+z^2)\ .$$

By separating variables in the time-independent Schrödinger equation as $\psi(x,y,z) = f(x)g(y)h(z)$ show that the energy levels have the form $\left(n+\frac{3}{2}\right)\hbar\omega$ where $n$ is a non-negative integer, and find their degeneracies.

Show that the ground state wave function is spherically symmetric.

5. A particle moves in two dimensions under the influence of the potential

$$V(x,y) = \frac{1}{2}m\omega^2\left(10x^2+12xy+10y^2\right)\ .$$

By considering $V$ in the rotated coordinates $u = (x+y)/\sqrt{2}$, $v = (-x+y)/\sqrt{2}$, find the energy levels and the associated degeneracy of each level.

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