O1 History of Mathematics
HT 2018 reading course: Nineteenth-Century British Symbolical Algebra

Christmas Vacation Reading

The subject of this year's reading course is the so-called *symbolical algebra* that we met briefly during the lecture course.

At the end of the eighteenth century, a small but vocal group of English mathematicians began to criticise the unrestricted use of negative numbers in algebra: they argued that negative numbers can only be understood by analogy, and that any idea that can only be grasped in this way is not a fit subject for teaching in schools. They thus called for the reconstruction of algebra without negative numbers. Complex numbers, it need hardly be said, were considered to be even further beyond the pale, as instances of where algebra simply does not work. Chief amongst these activists was the social reformer and religious non-conformist William Frend (1757–1841), who espoused the abandonment of negative numbers in his *Principles of Algebra* of 1796.

Most practicing mathematicians in England at this time, however, were far more practical and were not prepared to abandon negative numbers on such flimsy philosophical objections, certainly not as the more powerful continental approaches to calculus were finally being adopted in England in the early decades of the nineteenth century. Nevertheless, the criticism of negative numbers did not go away, and so a response to these was sought. This appeared in the *Treatise of Algebra* published in 1830 by the Cambridge don and Dean of Ely, George Peacock (1791–1858). Peacock's approach was to distinguish between so-called 'arithmetical algebra' on the one hand, and 'symbolical algebra' on the other. In arithmetical algebra, the symbols that are operated upon are assumed always to be positive integers, and therefore care is needed never to attempt to subtract a larger number from a smaller — Peacock's 'arithmetical algebra' is thus algebra in the restricted style promoted by Frend. In 'symbolical algebra', the emphasis is placed upon the rules of operation, rather than the interpretation of the symbols. Thus, thought Peacock, the objections raised by Frend could be bypassed.

Symbolical algebra, however, came in for various criticisms itself: for instance, if we do not know what the symbols mean, how can we operate on them? How can the results that we deduce about our 'empty' symbols have any objective truth? A prominent contributor to this debate was Augustus De Morgan (1806–1871). Initially sceptical about this approach to algebra, he later embraced it enthusiastically, before settling for a rather more pragmatic attitude: we may place the emphasis on the rules of operation in algebra provided the symbols operated on and the results obtained are, ultimately, susceptible to a concrete interpretation, even if that interpretation may not be imposed at every stage of a calculation. In general, however, British mathematicians did not take to this new approach to algebra, and it slowly faded from view. When abstract algebra (in the modern sense) appeared in Britain decades later, it did so as a continental import.

In this reading course, we consider a range of extracts (to be specified later on) from the writings of Frend, Peacock, and De Morgan, focusing on, but not limited to, the texts noted in the paragraphs above. As during the lecture course, the emphasis will be on the use of *original sources*. It should be emphasised that the recommended reading that will appear on the O1 course page will represent the bare minimum of reading needed: you will be expected to uncover further material for yourselves, which will then be the subject of discussion during our classes.

As preparation for the reading course, please read biographical material on the various figures mentioned above. A good starting point for this is the MacTutor History of Mathematics Archive <http://www-history.mcs.st-and.ac.uk/>, which features short biographies of mathematicians. If you scroll down to the bottom of each biography, you will find a link to further published biographies and other relevant secondary sources, many of which are available electronically through SOLO. You should go beyond the basic MacTutor biographies and explore the available material, particularly if certain parts of it grab your interest. We will discuss and compare the sources you have found in the first class of Hilary Term.

Finally, as practice in locating online resources, and as practical preparation for the reading ahead, you should track down the main primary sources that we will be using throughout the reading course, namely

- William Frend, *Principles of Algebra*, London, 2 vols., 1796, 1799;
- George Peacock, *A Treatise of Algebra*, Cambridge, 1830; 2nd ed. in 2 vols., 1842, 1845;
- Augustus De Morgan, *Trigonometry and Double Algebra*, London, 1849.

All of them are available on SOLO and/or Google Books.