Note: These problems are for practice and revision purposes. This sheet is not to be turned in.

1. Consider the wave equation
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$
subject to $u(x,0) = u(x,a) = 0$. Solve via separation of variables.

2. Solve Laplace's Equation, with boundary conditions as shown:
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ for } x \in (0,\pi),\ y \in (0,\pi),$$
$$u(0,y) = 0, u(\pi,y) = 0,$$
$$u(x,0) = 0, u(x,\pi) = \sin^3 x.$$

3. Solve the Heat Equation, with boundary conditions as shown:
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ for } x \in (0,\pi),\ t > 0,$$
$$u(x,0) = \sin^3 x, \quad u(0,t) = u(\pi,t) = 0\ \ \forall\, t > 0,$$
$$u \to 0 \quad \text{as } t \to \infty.$$

4. Recall: the Fourier transform of u is defined by
$$\hat{u}(t,k) = \int_{-\infty}^{\infty} u(x,t)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x, \tag{1}$$
and u may be recovered from $\hat{u}$ by using the *inversion formula*
$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{u}(t,k)\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}x. \tag{2}$$
Consider the heat equation
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \tag{3}$$
subject to $u \to 0$ as $x \to \pm\infty$ and $u = u_0(x)$ when $t = 0$. Solve by taking a Fourier transform in x.

5. Consider the boundary value problem
$$Ly(x) = f(x) \quad \text{on } 0 < x < 1, \qquad y'(0) + y(0) = \alpha, \quad y(1) = \beta, \tag{4}$$
with L is the differential operator given by
$$Ly \equiv y''(x) + 4y(x), \tag{5}$$
Derive a problem for the Green's function $g(x,\xi)$ in terms of the delta function $\delta(x)$ and give the form of the solution in terms of g.