

Alt. view of Green's function for ODE BVP

Ex $Ly \equiv y''(x) = f(x), \quad y(0) = \alpha, \quad y(1) = \beta$

Define $L^*v \equiv v'' \Rightarrow (Lu)v - u(L^*v) = u''v - v''u$
 $= \frac{d}{dx}(u'v - uv')$

Now $\int_0^1 dx \Rightarrow$ the eqn $(Ly)G - yL^*G$

$$\rightarrow \int_0^1 \underbrace{y''}_{f} G - y G'' dx = \underbrace{(y'G - yG')}_{\downarrow}$$

y' unknown on boundary, so
choose $G=0$ on $x=0,1$

If we choose $G(x,\xi)$ to satisfy

$$\begin{cases} G'' = \delta(x-\xi) \\ G(0,\xi) = G(1,\xi) = 0 \end{cases}$$

we get the soln

$$y(\xi) = \int_0^1 G(x,\xi) f(x) dx + \beta G'(1,\xi) - \alpha G'(0,\xi)$$