B4.3 Distribution Theory and Fourier Analysis: An Introduction MT18

General Prerequisites: Part A *Integration* is desirable. *Integral Transforms* and *Introduction to Manifolds* are useful but not essential.

Course Overview: Distribution theory can be thought of as the completion of differential calculus, just as Lebesgue integration theory can be thought of as the completion of integral calculus. It was created by Laurent Schwartz in the 20th century, not long after Lebesgue's integration theory.

Distribution theory is a powerful tool that works very well in conjunction with the theory of Fourier transforms. One of the main areas of applications is to the theory of partial differential equations. In this course we give a brief introduction to these three theories.

In addition to the Lecture Notes for the course I strongly recommend the book

[1] R.S. Strichartz: *A Guide to Distribution Theory and Fourier Transforms* (World Scientific, 1994. Reprinted: 2008, 2015)

Further Reading: The following books are more advanced, but also highly recommended for further reading and study.

[2] L.C. Evans, *Partial Differential Equations* (Amer. Math. Soc. 1998)
[3] E.H. Lieb and M. Loss, *Analysis* (Amer. Math. Soc. 1997)
[4] E.M. Stein and R. Shakarchi, *Fourier analysis. An introduction* (Princeton Univ. Press 2003)

A word of warning about notation: In this course we define the Fourier transform of an integrable function $f\colon \mathbb{R}^n \to \mathbb{C}$ by

$$\hat{f}(\xi) = \int_{\mathbb{R}^n} f(x)\mathrm{e}^{-\mathrm{i}x\cdot\xi}\,\mathrm{d}x.$$

This is in accordance with the definition you might have encountered in Part A options. However, it differs sligthly from the definition used in the book [1] where it is defined as

$$\hat{f}^S(\xi) = \int_{\mathbb{R}^n} f(x)\mathrm{e}^{\mathrm{i}x\cdot\xi}\,\mathrm{d}x.$$

The relation between the two definitions is obviously $\hat{f}(\xi) = \hat{f}^S(-\xi)$. In other books you will see yet other variants such as for instance

$$\int_{\mathbb{R}^n} f(x)\mathrm{e}^{-2\pi\mathrm{i}x\cdot\xi}\,\mathrm{d}x$$

and the like. This is annoying, but once you pay attention it is not hugely so, and the changes to formulae is not too difficult to keep track of.