B1.1 Logic

MT 18

Problem Sheet $\sharp$ 2

(1) Prove that for any $\phi, \phi_i, \psi, \chi \in \text{Form}(\mathcal{L})$ and any $\Gamma \subseteq \text{Form}(\mathcal{L})$
(i) $((\phi \to \psi) \to ((\phi \to (\psi \to \chi)) \to (\phi \to \chi)))$ is a tautology
(ii) $\Gamma \cup \{\psi\} \models \phi$ if and only if $\Gamma \models (\psi \to \phi)$
(iii) $\neg \bigwedge_{i=1}^{n} \phi_i$ is logically equivalent to $\bigvee_{i=1}^{n} \neg\phi_i$.
(iv) $(\phi \vee \psi)$ is logically equivalent to $((\phi \to \psi) \to \psi)$.

(2) Let ϕ be the formula $((p_0 \to p_1) \wedge (p_1 \to p_2))$. Find a formula in dnf logically equivalent to ϕ which is a disjunct of just three conjunctive clauses.

(3) (i) Prove that every formula is logically equivalent to one in conjunctive normal form.
(ii) Let v_0 be the valuation that assigns the value T to every propositional variable. Prove that a formula ϕ is logically equivalent to one built up from propositional variables using just the connectives $\wedge$ and $\to$ if and only if $\widetilde{v}_0(\phi) = T$.

(4) (i) Find the truth tables for all binary connectives $\star$ having the property that $\{\star\}$ is adequate. Justify your answer.
(ii) Show that there is no adequate unary connective.

(5) Prove that for any formulas α, β of $\mathcal{L}_0$, the following formulas are theorems of the system L_0. You may use the deduction theorem and, that for any α, β,

$$\text{if } \vdash (\alpha \to \beta) \text{ and } \vdash (\neg\alpha \to \beta) \text{ then } \vdash \beta.$$

(i) $(\neg\alpha \to (\alpha \to \beta))$
(ii) $(\neg\neg\alpha \to \alpha)$
(iii) $(\alpha \to \neg\neg\alpha)$
(iv) $((\neg\alpha \to \alpha) \to \alpha)$