Numerical Solution of Differential Equations II. QS 5

Question 1.

- Revisit Chapter 3 from LeVeque's book.
- Define in your words a conservative method.
- What is the benefit of using conservative methods?
- Is the entropy solution guaranteed using conservative methods?

Question 2.

Consider the Burgers equation in quasilinear form

$$u_t + uu_x = 0. \tag{1}$$

As observed during the lecture, assuming $u_j^n \geq 0$ for all j, n, the method

$$u_j^{n+1} = u_j^n - \frac{k}{h} u_j^n (u_j^n - u_{j-1}^n) \tag{2}$$

converges for smooth solutions.

- Implement in Matlab the scheme (2), and compute the numerical solution using the discontinuous initial condition mentioned in the lecture

$$u(x,0) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases}. \tag{3}$$

  Solve (1) up to $T = 0.5$ in the spatial domain $(-1, 1)$, using $k = 0.5h$, $h = 0.01$.
- Modify the initial condition to be

$$u(x,0) = \begin{cases} 1.2 & x < 0 \\ 0.4 & x \geq 0 \end{cases}, \tag{4}$$

  and compare the solution with the exact one (found using e.g. the method of characteristics).

Question 3.

A generalization of the Lax-Friedrichs scheme to Burgers equation (now written as $u_t + (\frac{1}{2}u^2)_x = 0$) is

$$u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{k}{2h}(f(u_{j+1}^n) - f(u_{j-1}^n)). \tag{5}$$

- Implement (5) using the initial conditions (3),(4).
- Does the computed solution converges to a weak solution?
- Is this the entropy solution?
- Write (5) in conservation form.