0.1 B5.6: Nonlinear Systems —Dr. Georgy Kitavtsev

Level: H-level **Method of Assessment:** Written examination.
Weight: Unit
Recommended Prerequisites: Part A Differential Equations 2.

Overview The course content was developed by Prof. Alain Goriely in several previous years. This course aims to provide an introduction to the tools and concepts of dynamical systems theory which have become a central tool of both pure and applied mathematics with applications in celestial mechanics, mathematical biology, fluid dynamics, granular media, and social sciences.

The course will focus on the geometry of both ordinary differential equations and maps. It will draw examples from appropriate model systems and various application areas. The problem sheets will require basic skills in numerical computation (numerical integration and visualisation of solutions of differential equations).

Learning Outcomes Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of dynamical systems. They will also get some exposure to some modern research topics in the field.

Synopsis

1. Geometry of linear systems

   Basic concepts of stability and linear manifolds of solutions. Orbits in phase-space, linear flows, eigenvalues of fixed points.

2. Geometry on nonlinear systems

   Notion of flows, invariant sets, asymptotic sets, attractor. Conservative systems and gradient flows.

3. Local analysis

   Lyapunov and asymptotically stable fixed points, Lyapunov functions, stable manifold theorem, notion of hyperbolicity, centre manifold.

4. Bifurcation.

   Bifurcation theory: codimension one normal forms (fold, pitchfork, transcritical, Hopf)

5. Maps

   Poincaré sections and first-return maps. Stability and periodic orbits; bifurcations of one-dimensional maps, period-doubling.

6. Chaos

   Maps: Logistic map, Bernoulli shift map, symbolic dynamics, Smale's Horseshoe Map. Melnikov's method
   Differential equations: Lorenz equations, Duffing equation.

Reading Students are by no means expected to read all these sources. These are suggestions intended to be helpful. The primary suggested reference is the book by Lawrence Perko.

1. L. Perko, *Differential Equations and Dynamical Systems* (Second edition, Springer, 1996).

2. Y. A. Kuznetsov, *Elements of Applied Bifurcation Theory* (Second edition, Springer, 1998).

3. S. H. Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering* (Westview Press, 2000).

4. R. H. Rand, *Lecture Notes on Nonlinear Vibrations.* [Available for free online at http://audiophile.tam.cornell.edu/randdocs/nlvibe52.pdf]

5. P. G. Drazin, *Nonlinear Systems* (Cambridge University Press, Cambridge, 1992).

6. Guckenheimer and Holmes, *Nonlinear Oscillations, Dynamical Systems* (Springer, 1983).