B5.6: Nonlinear Systems

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Overview The course content was developed by Prof. Alain Goriely. He also kindly provided the online written lecture notes Parts 1-3. The typed lecture notes Parts 4-5 were written by Dr. Georgy Kitavtsev. The present edition of the notes Parts 1-5 should be treated as a draft, they should not be considered as yet fully edited or finished. Most of the tutorial exercises and their solutions for this course were developed by Alain Goriely and James Kwiecinski.

The lecture notes posted online only cover the lectures. For students who want some more details the following books are recommended:

[S] Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering* (Westview Press, 2000).

[GH] Guckenheimer and Holmes, *Nonlinear Oscillations, Dynamical Systems* (Springer, 1983).

[P] Perko, *Differential Equations and Dynamical Systems* (Second edition, Springer, 1996).

[K] Kuznetsov, *Elements of Applied Bifurcation Theory* (Second edition, Springer, 1998).

Below, I link the subjects seen in class to the different books. The Perko seems to have the best examples.

Synopsis

1. Linear Systems

   1.1 Fundamental theorems. [P16], Examples in the plane [P20,S145]

   1.2 Stability theory [P39]

2. Nonlinear Systems

    2.1 Fundamental theorems [P70]

    2.2 Flow, asymptotic behaviour [P87,95]

    2.3 Stability theory, Lyapunov function [P129]

3. Local analysis

    3.1 Stable manifold theorem [P105]

    3.2 Centre manifold [P153]

    3.3 Reduction to centre manifold [P153]

    3.4 Mappings [S348]

4. Bifurcations

    4.1 Topological equivalence and structural stability [P316,K39]

    4.2 Local bifurcations [P311,S241,K80]

    4.3 Sotomayor's theorem [P330]

    4.4 Hopf Bifurcation [S248,P293,K86]

    4.5 Bifurcations of 1D maps [GH156,K113]

    4.6 Logistic map [S353]

5. Chaotic dynamical systems

    4.1 Symbolic dynamics [P316,K39]

    4.2 Melnikov's method for planar autonomous systems [K229]

    4.3 Melnikov's method for periodically perturbed planar systems [P415,GH184]

    4.4 Smale's horseshoe map [P409,GH230]