

C6.3 Approximation of Functions

Michaelmas Term 2017

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Lecture times: Wednesdays 9:00 - 10:00 in L4. Thursdays 9:00 - 10:00 in L3.

Tutorials: The lecturer is also the tutor. Meeting times and locations:

Group 1: Thursdays 15:00-16:00, weeks 3 – 8, L4. (Week 6 tutorial moved to Nov 21, 3pm, C1.)

Group 2: Tuesdays 17:00-18:00, weeks 3 – 8, L5.

Scope: This course is aimed at Part C (4th year) Mathematics students (C6.3) and students in the MSc in Mathematical Modelling and Scientific Computing. It describes techniques for how to approximate a function drawn from an infinite dimensional space (say, the Lipschitz continuous functions on an interval) by finding another function in a finite dimensional space (say, polynomials of some given maximal degree) that is in some sense “close”. This material forms a foundation for much of numerical analysis and scientific computing, and we will explore connections to methods for numerical integration and differentiation, for solving differential equations, etc.

Topics covered include: Chebyshev interpolants, polynomials, and series. Barycentric interpolation formula. Weierstrass approximation theorem. Convergence rates of polynomial approximations. Hermite integral formula and Runge phenomenon. Lebesgue constants, polynomial rootfinding. Orthogonal polynomials. Clenshaw-Curtis and Gauss quadrature. Rational approximation.

Text: The course will closely follow the textbook *Approximation Theory and Approximation Practice*, SIAM, 2013, <http://www.maths.ox.ac.uk/chebfun/ATAP/>.

Problem sheets: For each tutorial, a problem sheet will be due the Monday before at 12:00 noon. The problems are the following exercises from the textbook:

- (1) Due 12:00 Mon 23 Oct, week 3: 1.3, 2.1, 2.2, 2.4, 2.5, 3.5, 3.9.
- (2) Due 12:00 Mon 30 Oct, week 4: 4.1, 4.7, 5.6, 5.7, 5.8, 5.9, 6.2, 6.3.
- (3) Due 12:00 Mon 06 Nov, week 5: 7.1, 7.2, 7.5, 7.6, 8.2, 8.4, 8.7, 8.10.
- (4) Due 12:00 Mon 13 Nov, week 6: 9.2, 9.8, 10.2, 10.3, 10.4, 10.5, 10.6, 11.3.
- (5) Due 12:00 Mon 20 Nov, week 7: 13.1, 13.2, 13.5, 14.1, 14.2, 15.4, 15.9.
- (6) Due 12:00 Mon 27 Nov, week 8: 17.3, 17.5, 17.8, 18.4, 18.5, 18.7, 19.6, 19.9.

Assessment: For Part C students, by exam in Trinity Term. For MSc students, by Special Topic, due Monday of week 1, Hilary Term (with great flexibility on topics to write about).

Matlab and Chebfun: We will make constant use of Chebfun (www.chebfun.org), which is built on Matlab. Each problem sheet contains a mix of theory and computation. The examination will involve no Matlab or Chebfun, so in principle you can make it through the course without paying attention to computing, but you would be unlikely to fully learn the material this way.

Access to MATLAB and Chebfun: Matlab is available from

<https://register.it.ox.ac.uk/self/software>

To get Chebfun, go to

www.chebfun.org

and click on Download, which will give you two lines of text you can paste into your Matlab command window. Then all you need to do is to make sure Chebfun is in your Matlab path. If you prefer, you can download Chebfun from GitHub.

Course web page: <https://courses.maths.ox.ac.uk/node/4908>