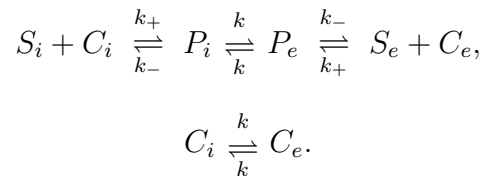


## Mathematical physiology

### PROBLEM SHEET 1.

- Carrier-mediated transport of a substrate  $S$  by a carrier protein  $C$  is modelled as the (rapid) reaction system



Explain the meaning of these reactions. If a substrate flux  $J$  is supplied to the extra-cellular fluid and thus also (in a steady state) to the intra-cellular fluid, use steady state kinetics to show that

$$J = \frac{K^*(S_e - S_i)}{(K_m + S_i)(K_m + S_e) - K_d^2}, \quad K_m = \frac{k_- + k}{k_+}, \quad K_d = \frac{k}{k_+},$$

where  $K^*$  should be defined.

- A membrane channel has  $N$  identical gates. If  $S_i$  is the proportion of channels with  $i$  open gates, write down rate equations for  $S_i$  in terms of the overall reaction rates  $R_i$  of  $S_{i-1} \rightleftharpoons S_i$ ,  $i = 1, 2, \dots, N$ . Derive a conservation law expressing the conservation of the total number of channels.

Suppose that

$$S_j = {}^N C_j n^j (1 - n)^{N-j},$$

where  ${}^N C_j$  is the binomial coefficient. Show that the equations are satisfied if

$$\dot{n} = \alpha(1 - n) - \beta n, \quad (*)$$

where  $\alpha$  and  $\beta$  are the gate opening and closing rates.

For the case  $N = 2$ , show that all initial states tend to this solution (*put*  $S_0 = (1 - n)^2 + y_0$ ,  $S_2 = n^2 + y_2$ , *where*  $n$  *satisfies*  $(*)$ , *and show that*  $y_0, y_2 \rightarrow 0$ .)

- [*May not be covered in class.*] Suppose a membrane channel has three gates, two of which are controlled by a protein  $M$ , and the other is controlled by a protein  $H$ . Suppose that the fractions of open  $M$  and  $H$  gates are  $m$  and  $h$  respectively. By letting  $S_{ij}$  denote the density of channels with  $i$  open  $M$ -gates and  $j$  open  $H$ -gates, write down the rate equations for  $S_{ij}$ , assuming that the rates of  $M$ -gate opening and closing are  $\alpha$  and  $\beta$ , and the rates of  $H$ -gate opening and closing are  $\gamma$  and  $\delta$ , respectively.

Show that the equations have solutions in which  $S_{00} = (1 - m)^2(1 - h)$ , etc., providing  $m$  and  $h$  satisfy equations which you should find. [*There is no need to be exhaustive.*]