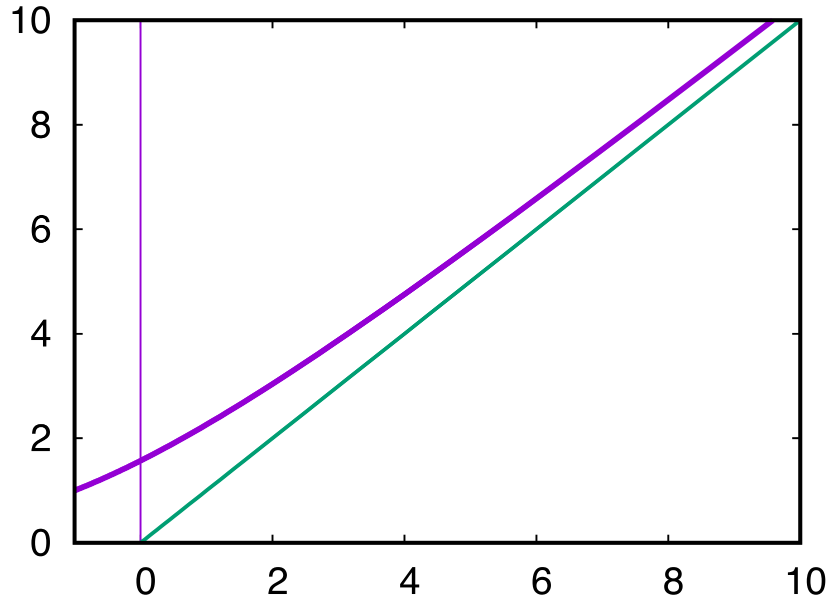

Figure 1: $\Gamma(\alpha)$ defined by (1), together with its asymptote, $\Gamma = \alpha$.

Plots for question 4.5

The curve defined defined parametrically (see figurer 1) by

$$\Gamma(\alpha) = \frac{\Omega}{\sin\Omega}, \quad \alpha = -\frac{\Omega}{\tan\Omega}, \tag{1}$$

for $\Omega \in (0, \pi)$ has the series solution

$$\Gamma \approx 1 + 0.5(\alpha+1) + 0.075(\alpha+1)^2 - 0.0032(\alpha+1)^3 + O[(\alpha+1)^4]. \tag{2}$$

Figure 2 shows the quadratic approximation

$$\Gamma \approx 1 + 0.5(\alpha+1) + 0.058(\alpha+1)^2; \tag{3}$$

figure 3 shows the cubic approximation

$$\Gamma \approx 1 + 0.5(\alpha+1) + 0.075(\alpha+1)^2 - 0.005(\alpha+1)^3. \tag{4}$$

Efforts to find a value c such that the quartic approximation

$$\Gamma \approx 1 + 0.5(\alpha+1) + 0.075(\alpha+1)^2 - 0.0032(\alpha+1)^3 + c(\alpha+1)^4 \tag{5}$$

provides an improved approximation for larger values of α do not fare so well (figure 4.

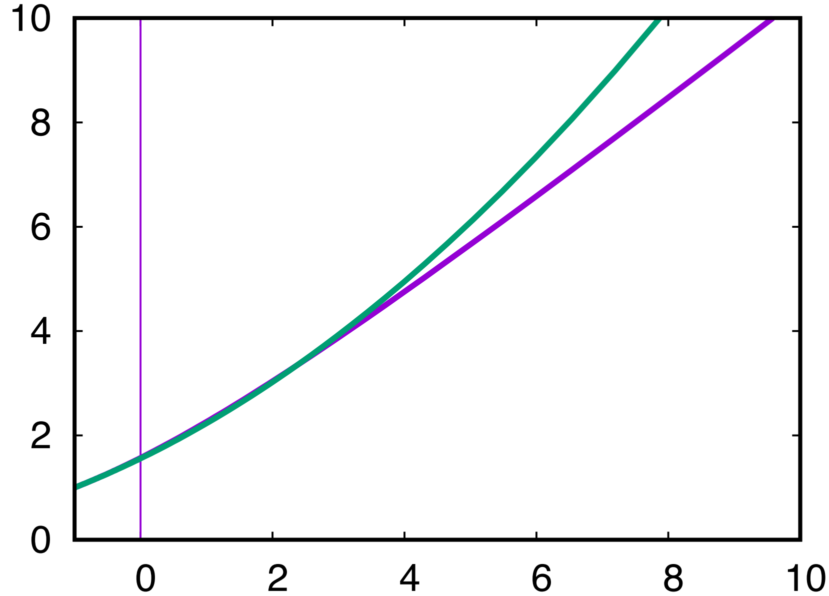

Figure 2: Quadratic approximation given by (3).

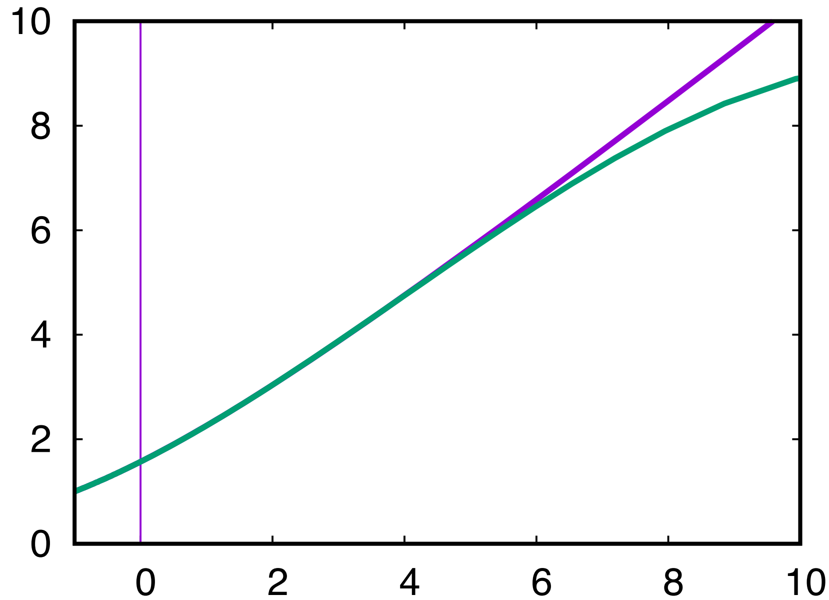

Figure 3: Cubic approximation given by (4).

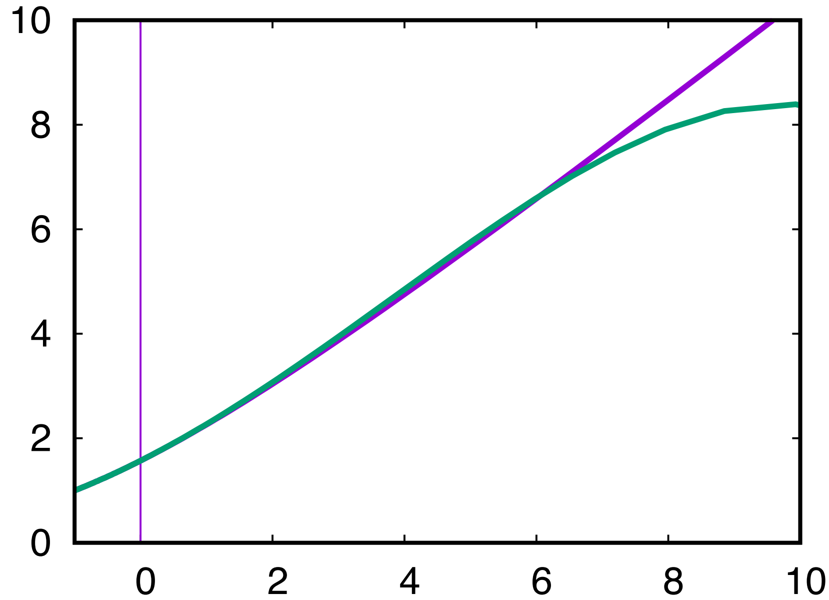

Figure 4: A plot of (5) using a value $c = -0.0002$. While the approximation is slightly improved for $\alpha > 5$, it is less good at smaller α.

The rational approximation given by

$$\Gamma = \frac{1 + b(\alpha+1) + c(\alpha+1)^2}{1 + c(\alpha+1)}, \tag{6}$$

with $b = 0.65$ and $c = 0.15$, is shown in figure 5. With $b = 0.69$ and $c = 0.3$ (figure 6, the approximation is essentially perfect. (The maximum error for $\alpha < 100$ is less than 0.05.)

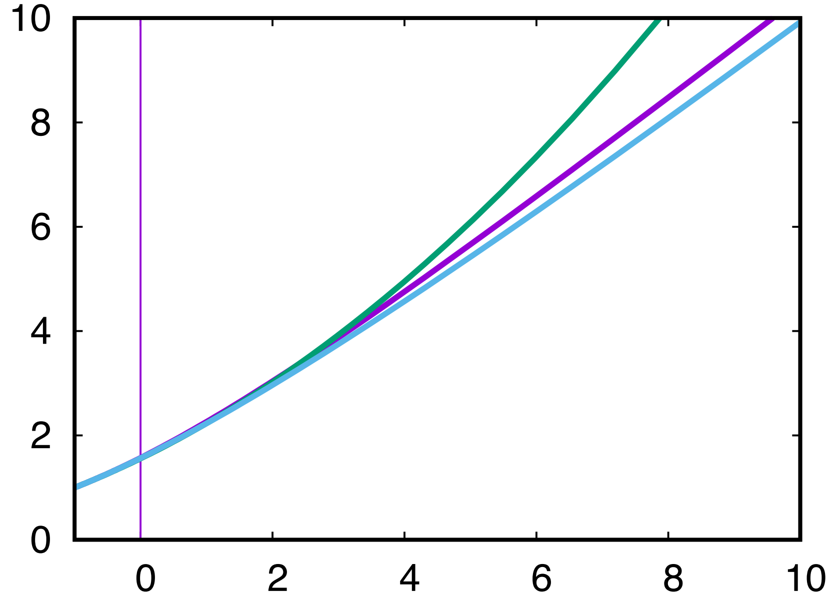

Figure 5: The rational approximation (6), with $b = 0.65$, $c = 0.15$.

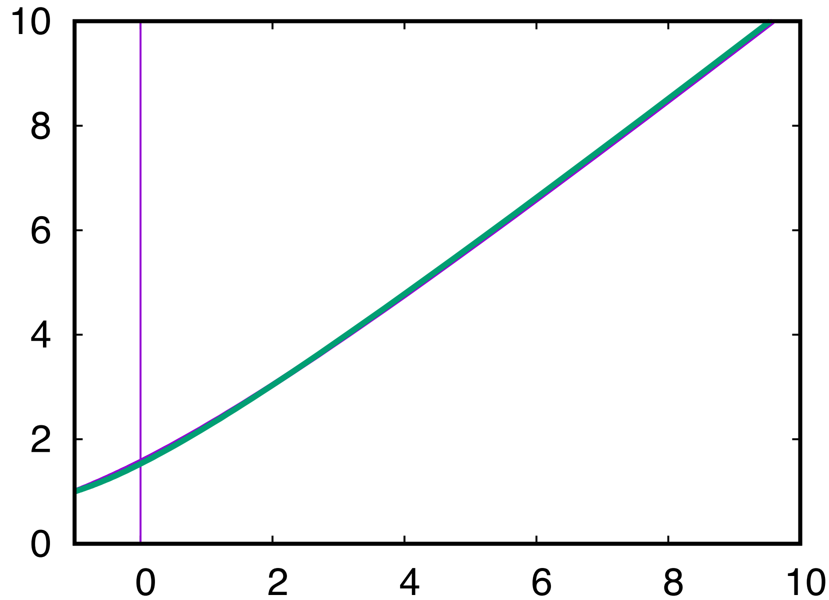

Figure 6: The rational approximation (6), with $b = 0.69$, $c = 0.3$.