Assessment as a broadening course

Note: This is not relevant to undergraduates.

Some DPhil students doing this lecture course may want to submit it as a 'broadening course', and so need to be assessed on it.

If so, you can do one of the miniprojects below. I am expecting you to produce an essay-type answer, of length perhaps 5 pages, ideally written in LaTeX and submitted as a PDF file by e-mail to `joyce@maths.ox.ac.uk`, by Monday 14th January 2019. You could spend perhaps three days on the project, of which half might be reading references. Don't waste your time; from the point of view of assessment, all you have to demonstrate is that you've learnt something and understood it, and can string a sentence together.

These are just suggestions; feel free to choose your own topic.

Project 1. Define connections ∇ on a vector bundle $E \to X$, and the curvature of ∇. Explain why a Riemannian manifold (X, g) has a natural connection ∇ on TX, the Levi-Civita connection. Discuss Riemann curvature. Give some idea of reasons why it is important, e.g. General Relativity.

Project 2. Give an introduction to the theory of Lie groups and Lie algebras.

One good book (there are several) is R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie algebras*, LMS, 1995.

Project 4. Explain the proof of de Rham's theorem of the isomorphism between de Rham cohomology of a manifold and sheaf cohomology over $\mathbb{R}$, using sheaf cohomology and fine sheaves.

Project 5. Discuss Hodge theory for compact, oriented Riemannian manifolds; the isomorphism between de Rham cohomology and the vector spaces of harmonic forms. Include Poincaré duality.

Project 6. Write about spin geometry for Riemannian manifolds: Clifford algebras and the spin representation, spin structures on a manifold, spin bundles and spinors, the Dirac operator. Could mention quantum theory, relevance of spinors to spin $\frac{1}{2}$ particles like electrons.

Project 7. Give a broad-brush account (not much detail necessary) of some milestones in the theory of 3-manifolds: the Poincaré Conjecture and its proof by Perelman (and hangers-on); perhaps also describe Thurston's Geometrization Programme.