$\boxed{1}$

radial space-like geodesics

(1) $\qquad E = F\dot{t} \qquad\qquad F = 1 - \frac{2M}{r}$

(2) $\qquad 2L = \underset{\substack{\uparrow \\ \text{SL geodesic}}}{1} = -F\dot{t}^2 + F^{-1}\dot{r}^2$

(1) into (2) to eliminate $\dot{t}$

$$1 = -\cancel{F}\frac{E^2}{F^{\cancel{2}}} + F^{-1}\dot{r}^2$$

$$F = -E^2 + \dot{r}^2 \implies \underline{\dot{r}^2 = E^2 + F}$$

$$= E^2 + 1 - \frac{2M}{r}$$

let $\quad E^2 + 1 = \frac{2M}{r_0}$

ie $\quad r_0 = \frac{2M}{E^2+1} < 2M$

Then $\quad \dot{r}^2 = 2M\left(\frac{1}{r_0} - \frac{1}{r}\right)$

Also (1) $\iff \quad \dot{t} = E\left(1 - \frac{2M}{r}\right)^{-1}$

let $E = 0$: then $r_0 = 2M$

and $\dot{t} = 0$ ie $t = \text{constant}$

In terms of u & v

$t = \text{constant}$ surfaces correspond to lines $uv^{-1} = \text{constant}$

Note that $\dot{r}^2 = F = 1 - \frac{2M}{r}$

so r decreases to $r = r_0 = 2M$ and then increases again

radial time-like geodesics:

$(1)'$ $E = F\dot{t}$

$(2)'$ $2L = -1 = -F\dot{t}^2 + F^{-1}\dot{r}^2$

$\uparrow$ TL-geodesics

$(1)'$ into $(2)'$ to eliminate $\dot{t}$

$$-1 = -\frac{E^2}{F} + \frac{1}{F}\dot{r}^2$$

$$\Rightarrow \quad \dot{r}^2 = -F - E^2 = +E^2 - 1 + \frac{2M}{r}$$

E=0 : $\dot{t} = 0$ so $t = \text{constant}$

$$\dot{r}^2 = -1 + \frac{2M}{r} \geq 0 \quad \text{so} \quad r \leq 2M$$

r increases to $r = 2M$ and then decreases again

E<1 : let $E^2 - 1 = -\frac{2M}{r_0}$

so $r_0 > 2M$

Then $$\dot{r}^2 = 2M\left(\frac{1}{r} - \frac{1}{r_0}\right) \qquad r < r_0$$

$$\dot{t} = E\left(1 - \frac{2M}{r}\right)^{-1}, \quad E^2 = 1 - \frac{2M}{r_0}$$

r increases to $r = r_0 > 2M$

and then decreases again