C5.6 Applied Complex Variables

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0.1 Recommended Prerequisites

The course requires Part A core complex analysis, and is devoted to extensions and applications of that material. A knowledge of the basic properties of the Fourier transform, as found for example in Part A *Integral Transforms*, will be assumed. Part A *Fluid Dynamics and Waves* is helpful but not absolutely essential: the necessary results from inviscid two-dimensional hydrodynamics will be quoted as required, and for further details the reader is referred to the Part A lecture notes. Part C *Perturbation Methods* is also helpful in the analysis of certain contour integrals.

0.2 Synopsis

Review of core complex analysis, especially analytic continuation, multifunctions, contour integration, conformal mapping and Fourier transforms. [3 lectures, problem sheet 1]

Riemann mapping theorem (statement only). Schwarz–Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain. Applications to inviscid hydrodynamics: flow past an aerofoil and other obstacles by conformal mapping. [2 lectures, problem sheet 2]

Steady and unsteady free surface flows. [3 lectures, problem sheet 2]

Applications of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Riemann-Hilbert problems. Cauchy singular integral equations. [3 lectures, problem sheet 3]

Transform methods, complex Fourier transform. Contour integral solutions of ODEs. [2 lectures, problem sheet 4]

Wiener-Hopf method. [3 lectures, problem sheet 4]

0.3 Reading list

[1] G. F. Carrier, M. Krook and C. E. Pearson, *Functions of a Complex Variable* (Society for Industrial and Applied Mathematics, 2005.) ISBN 0898715954.

[2] M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, Cambridge University Press, 2003). ISBN 0521534291.

[3] J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movichan, *Applied Partial Differential Equations: Revised Edition* (Oxford University Press, 2003). ISBN 0198527713.