Math C5.4, Networks, University of Oxford
Problem Sheet 1

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1. *Introduction*

   (a) Download and install a programming interface of your choice, ideally MATLAB or Python notebooks. See `http://www.maths.ox.ac.uk/members/it/software-personal-machines/matlab` and `http://jupyter.org/install.html`

   (b) Practice with MATLAB. For example, you could go through parts of a tutorial available at `http://www.tutorialspoint.com/matlab/` or `http://www.tutorialspoint.com/python/`. There are also numerous other resources for learning MATLAB available online. In your homework submission, indicate what practice and tutorials you have done. Cite all references explicitly.

   (c) For Matlab, download the Brain Connectivity Toolbox from `https://sites.google.com/site/bctnet/`. If you use Python, download NetworkX from `https://networkx.github.io/`. This is one of many such sources available online.

   (d) Download data for at least two small unweighted, undirected networks of somewhat different sizes from `http://konect.cc/` and have a look at the data format. Import the graph by using the library of your choice (networkX, etc.).

   (e) Using some software, draw visualisations of these networks. Possibilities include visone and Gephi.

   (f) Plot the degree distribution for each of the two networks that you downloaded. What are you able to conclude from these degree distributions?

2. *Mathematical toolbox.*

   (a) Ex.III.1 : Using the computer language of your choice, calculate the mean and variance of a Bernouilli process, as a function of p.

   (b) Ex.III.4 : Take a RW on a discrete one-dimensional line. Assuming that the walker has, at each step, a probability 1/2 to go to the left and 1/2 to go to the right. Explore by numerical simulations how the probability distribution evolves over time, and verify the accuracy of Eq.(45). Provide a simple metric to test the "Gaussianity" of the distribution.

   (c) Ex.III.8 : Calculate numerically the distribution of eigenvalues of a random symmetric matrix A of size 1000, where each entry is an independent Bernouilli random variable, subject to the constraint that $A_{ij} = A_{ji}$ to ensure symmetry. Describe your observations.

3. *Connectedness.*

   (a) Draw a graph that is weakly connected but not strongly connected. Write down the adjacency matrix of this graph. What can happen to a random walk in such a graph, and what implication does this have for the asymptotic density of walkers on the nodes?

   (b) Consider the adjacency-matrix representation of a graph. What is the difference between the spectrum of directed networks versus that of undirected networks? (Recall that the set of eigenvalues of a matrix is called the *spectrum* of that matrix.)

4. *Clustering coefficients.* Draw a very small network in which the global clustering coefficient and mean local clustering coefficient have different values. Write down the adjacency matrix for this network.

5. *Small-world.* Ex.IV.4 : Generate graphs of the model c from Figure (12) and calculate the dependence of the network diameter and clustering coefficient on the number of shortcuts. Is this model, very similar to the so-called Watts-Strogatz model, a good model for a social network? Why or why not?

6. *Centrality measures.*

   (a) Ex.IV.5 : Take an undirected network and measure the correlation between different centrality measures. The correlation can either be estimated with the centrality values (Pearson) or with their associated ranking (Kendall). Construct an example of a graph where one node has a small degree centrality but a high betweenness centrality.

(b) Dynamical importance: Read "Characterizing the Dynamical Importance of Network Nodes and Links", PRL 2006, and derive its equations (4) and (5), emphasising the assumptions that you make.