Math C5.4, Networks, University of Oxford
Problem Sheet 4

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1. *Dynamics, time-scales and Communities*

   (a) Ex.VII.1 : Write a code to simulate consensus dynamics on a network, and verify that the dynamics asymptotically converges towards the state $x_* = \mathbf{1}^\top \mathbf{x_0}/n$.

   (b) Ex.VII.2 : Write a code to reproduce the numerical results of Figure 23 in the lecture notes

2. *Dynamics I: Random walks*

   (a) Ex.VIII.1: Write your own code to calculate the Pagerank of a directed network. Test on an example the dependency of Pagerank on the teleportation coefficient. Test your code on an undirected, connected, regular network and comment the results.

   (b) Ex.VIII.2: By performing stochastic simulations of an ensemble of random walkers on a graph, verify numerically Kac's formula.

3. *Random walks to reveal network structure*

   (a) Ex IX.1: Propose and justify a generalization of Markov stability Eq. 207 in the case of directed networks.

   (b) Ex IX.2: Read "Comparing clusterings - an information based distance, M Meila, 2017", and implement numerically a method to compare different partitions. Compare the partitions obtained by maximising modularity and the Map Equation on the network of your choice.

4. *Epidemic processes*

   (a) Read Nature of the Epidemic Threshold for the Susceptible-Infected-Susceptible Dynamics in Networks, Boguñá, Marian and Castellano, Claudio and Pastor-Satorras, Romualdo, Phys. Rev. Lett. 2013, and write a 1-page summary, typeset using LaTeX, of the main findings and methods of this article, as well as its strength and limitations.