Some very short hints

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Introduction

Only look at this after you spent at least 1 hours on the non-bookwork part of the question. The learning effect is a lot higher if you come up with a solution yourself.

In this document, I'll provide some very short hints for past Axiomatic Set Theory papers. These were made without referencing the model solutions and might in fact not work or be incorrect on closer inspection.

2018

Q1,c Consider V_γ for a limit γ.

Q1,d Consider aRb iff $b \in \omega \land \phi(b, a)$.

Q2,b,iii Show that the $\leq_L$-least bijection belongs to D and so does the pre-image of each element of $|x| \cap D$.

Q2,b,iv Consider the $\leq_L$-least witness for an existential formula.

Q2,b,v There are only countably many formulae.

Q3,b,i Consider 'there exists a surjection $\omega \to a$' and 'f is a surjection $\omega \to a$'.

Q3,b,ii To show that its a limit, consider 'there is a larger ordinal than α'.

Q3,b,iii Show that 'being a cardinal' is absolute for H_κ, L.

2017

Q1,c,i Note $a \in A \iff V \models \forall t\ t \notin a$.

Q1,c,ii $F = \emptyset$ and $F = \{a\}$ works.

Q1,c,iii Consider the union of the witnessing Fs.

Q1,c,iv Show $C = \{\{a, a'\} : a, a' \in A\} \in S_A$ and if f is a choice-function on C and distinct $a, a' \notin F$ then the permutation switching a and a' moves f.

Q2,d In the case $y \cap TC(x) \neq \emptyset$, note that if $x \in m \in TC(x)$ then $x \in TC(x)$ so $\{x\} \cup TC(x) = TC(x)$.

Q3,b,ii Note that 'λ is a cardinal' is absolute for H_κ, V and consider $n \mapsto \aleph_n$.

2016

Q1,b $\mathcal{P}(\alpha)^{\mathrm{On}} = \alpha + 1$

Q1,c Make sure that you only consider finitely many classes (e.g. by using a parameter for the recursion). Consider $\{\{1\}\}$.

Q2,b,weakly inaccessible $\implies \kappa = \aleph_\kappa$ Set $\kappa = \aleph_\gamma$, note $\gamma \leq \kappa$ and $cf(\aleph_\gamma) \leq \gamma$.

Q3,c,Pairing Consider $z = F^{-1}(\{x, y\})$.

Q3,c,Union Consider $z = F^{-1}\left(\bigcup_{t \in F^{-1}(x)} F^{-1}(t)\right)$.

Q3,c,Infinity Define the $n^{(V,E)}, n \in \omega$ recursively and consider $F^{-1}\left(\left\{n^{(V,E)} : n \in \omega\right\}\right)$.

Q3,d Consider F which swaps 0 and 1 and note that $(V, E) \models 0 = \{0\}$ which also satisfies ϕ.