Scientific Computing for DPhil Students II
Assignment 2

Due at lecture at 11:00 on Tuesday, 5 February 2019.

These problems are from *Exploring ODEs,* which is freely available online from `https://people.maths.ox.ac.uk/trefethen/`. Most of them can be solved in a few lines of Chebfun; be sure to include listings of your code. If you prefer to use Matlab, that is OK.

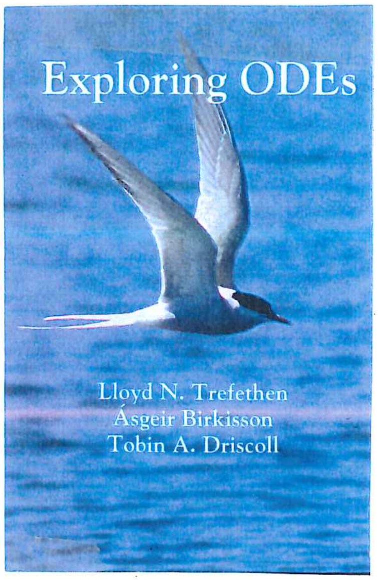
Exercise 8.1. Exploiting resonance to increase amplitude. For a given frequency ν, consider the solution to $y'' + y = 1 - \cos(\nu t)$, $y(0) = 1$, $y'(0) = 0$. *(a)* If ν is set equal to the resonant frequency ω for this equation, compute the time t_c at which $y(t)$ first reaches the value 10. *(b)* What is the smallest value of ν for which $y(t)$ reaches the value 10 at some time $t \in [0, 100]$?

Exercise 11.1. *Cleve Moler's favorite ODE.* Consider the IVP $(y')^2 + y^2 = 1$, $y(0) = 0$ with the additional constraint $-1 \le y \le 1$. *(a)* Show that there are exactly two solutions for $t \in [0, 1]$ and state formulas for them. *(b)* Show that there are infinitely many solutions for $t \in [0, 2]$.

Exercise 13.3. Alternative choices of the Lorenz coefficient 28. In the Lorenz equations, let r denote the parameter that traditionally takes the value 28. Starting from our usual initial conditions, make plots of $u(t)$ against $w(t)$ as in Figure 13.1 for $t \in [0, 100]$ with $r = 20, 22, 24$; also make plots in each case of $u(t)$ against t as in Figure 13.2. Which case seems to be chaotic? Which one gives the clearest example of transient chaos?

Exercise 13.4. Lorenz equations with a breeze. In the Lorenz equations (13.1), let the first equation be changed to $u' = 10(v - u) - a$, where a is a parameter. As in the last exercise, make plots of $u(t)$ against $w(t)$ and of $u(t)$ against t for $t \in [0, 30]$ with $a = 20, 25, 30$. Comment on the solutions.

Exercise 15.5. A cyclic system of three ODEs (adapted from Guckenheimer and Holmes, "Structurally stable heteroclinic cycles," *Mathematical Proceedings of the Cambridge Philosophical Society,* 1988). Consider the system of ODEs $u' = u(1 - u^2 - bv^2 - cw^2)$, $v' = v(1 - v^2 - bw^2 - cu^2)$, $w' = w(1 - w^2 - bu^2 - cv^2)$, where b and c are parameters. *(a)* Plot the solution $u(t)$ for $t \in [0, 800]$ with $b = 0.55$, $c = 1.5$ and initial conditions $u(0) = 0.5$, $v(0) = w(0) = 0.49$. Make similar plots of $v(t)$ and $w(t)$, and also of the whole trajectory in u-v-w space, and comment on these shapes. *(b)* What are the four fixed points of this system that the plots just drawn come close to? For large t, the trajectory moves approximately in a cycle from one fixed point, to another, to a third, and then back again. (It is approximating a *heteroclinic cycle.*) Which fixed point is the trajectory near at $t = 800$?



Exercise 16.1. Fisher equation. The function $y(x)$ satisfies $y'' + y - y^2 = 0$ for $x \in [-1, 1]$ with $y(-1) = 1$, $y(1) = 0$. *(a)* If $y(0) \approx 0.6$, what is $y(0.5)$? Plot the solution. *(b)* If $y(0) \approx -2.5$, what is $y(0.5)$? Plot the solution.