Scientific Computing for DPhil Students II
Assignment 4

Due at the Andrew Wiles Building reception at 11:00 on Tuesday morning of Week 8, 5 March 2019. No papers will be accepted after this hour, and solutions will be posted at this time. This is the last of four assignments this term.

Go to `courses.maths.ox.ac.uk/node/41040` and review the lecture notes and demonstration M-files for this course. Then compute the following six numbers, each to at least three digits of relative accuracy (i.e. 0.665 or 23.2 or 0.00277). For each problem, also make a plot of the solution.

You absolutely do not have to figure out the optimal method for any of these problems. What you do have to do is explain what you've done. However you get the number, explain your method plainly, including crucial code segments as necessary and making it clear what evidence you have for three digits of accuracy.

1. What is the smallest eigenvalue λ of the quartic Schrödinger problem $-u_{xx} + x^4 u = \lambda u$ with zero boundary conditions on the interval $[-5, 5]$?

2. The van der Pol equation

$$u'' - a(1 - u^2)u' + u = 0, \quad a > 0,$$

has oscillating periodic solutions with period $T = 10$. What is a?

3. At what time $t = t_c$ does the solution u to the Allen-Cahn equation

$$u_t = 0.015u_{xx} + u - u^3, \quad u(-1, t) = u(1, t) = -1, \ u(x, 0) = 1 - 2x^2$$

become negative?

4. The solution to the initial boundary-value problem

$$u_t = u_{xx} + u_x + e^u, \quad u(-1, t) = u(1, t) = 0, \ u(x, 0) = 0$$

blows up to ∞ at a finite time t_c. What is t_c?

5. The advection-diffusion equation $u_t = u_{xx} - 20u_x$ is posed with zero boundary conditions on $-4 \le x \le 4$ and initial data $1 - |x|$ for $|x| < 1$, zero otherwise. Let $m(t)$ be the point in $[-4, 4]$ at which $u(x, t)$ takes its maximum value. What is $\max_{t>0} m(t)$?

6. The heat equation $u_t = u_{xx} + u_{yy}$ is solved on the square domain $0 \le x, y \le 1$, with zero boundary conditions and initial conditions 1 for $0.25 \le x, y \le 0.75$, 0 otherwise. At what time t_c does $\max_{x,y} u(x, y, t)$ fall under 0.5?