Quantum Field Theory

Homework Assignment #3

due week 6, as arranged by your tutor

1. Consider the integral

$$\int d^4k \frac{e^{-ik\cdot x}}{k^2 - m^2}$$

There are four possible contours for k^0, depending on how the contour goes around the two singularities on the real k^0 axis. For each of these four cases, do the k^0 integral to express the integral in terms of $D(x-y)$ and $D(y-x)$, where

$$D(x) \equiv \langle 0|\phi_I(x)\phi_I(0)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}e^{-ik\cdot x} \ . \tag{1}$$

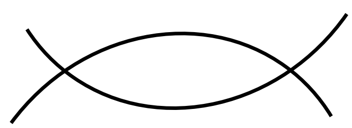
In which cases can one find an $i\epsilon$ prescription? Find the corresponding prescription. Explain which integral gives the retarded propagator

$$D_R \equiv \theta(x^0 - y^0)\langle 0|\left[\phi_I(x),\, \phi_I(y)\right]|0\rangle \ , \tag{2}$$

where $\theta(t)$ is the usual step function.

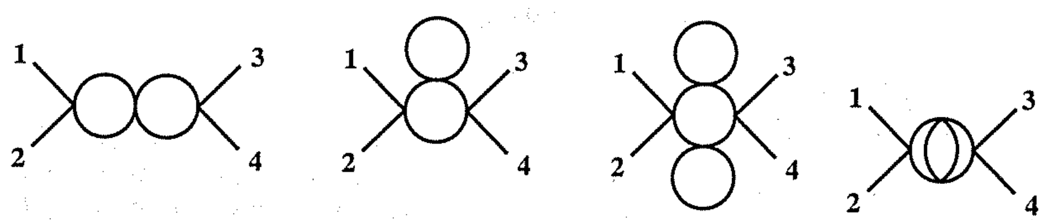
2. (Peskin 2.3) Evaluate $D(x-y)$ in terms of Bessel functions for $(x-y)$ spacelike. **Hint 1**: First work in the frame where $x^0 = y^0$, and then get the general answer by writing the result in terms of frame-independe quantities. **Hint 2**: My favourite book on special functions is by Abramowitz and Stegun, available for free on the web. Others like Gradshteyn and Ryzhnik.

3. Consider the following Feynman diagram in position space



Write down its evaluation in terms of various $D_F(x)$; do not use the explicit expression for D_F or attempt any integrals. Make sure you define all spatial variables precisely. Fourier transform your expression, again not using the explicit expression for D_F but just writing the answer in terms of $\widetilde{D}_F(p)$ for various p.

4. Consider the following Feynman diagrams for $2 \to 2$ scattering in scalar field theory:



(a) Find the symmetry factor for each, both by doing the combinatorics and by understanding the "symmetries" of each graph. Be honest – do them independently and then make sure they agree!

(b) Write down the scattering amplitudes for each Feynman diagram, i.e. amputate the external propagators. Make sure you define precisely your convention for the momenta. Get rid of any integrals you can using δ functions, but otherwise leave your amplitudes as integrals.

5. In a lecture I related quantum-mechanical perturbation theory to the field-theory version by drawing diagrams representing various intermediate states in scattering processes, possible because of particle production. This requires taking care with the time ordering, in order to define the intermediate states properly. I did this explicitly for the s channel to order μ^2 in a theory including a ϕ^3 interaction. Complete this relation by doing the same calculation for the t and u channels: write down the appropriate diagrams both in field theory and in the QM approach, and related the two results.

6. Consider two scalar fields with different masses and action

$$S = \frac{1}{2}\int d^4x \left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2 + \partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2 - \lambda_1\phi^4 - \lambda_2\Phi^4 - \lambda_3\Phi^2\phi^2 \right.$$
$$\left. + \mu_1\phi^3 + \mu_2\Phi^3 + \mu_3\phi\Phi^2\right) \; .$$

Treat each λ as the same order as any μ^2. Write down all Feynman diagrams for the $\phi\phi \to \Phi\Phi$ scattering process up to second order. Make sure to come up with some convention to distinguish ϕ and Φ lines. Do likewise for the $\Phi\Phi \to \Phi\Phi$ process. Now consider the case where $M > 2m$, so that the particle can decay. Write down the diagrams for $\Phi \to \phi\phi$ up to and including *third* order in the small parameters (i.e. up to μ^3 or $\lambda\mu$).