Private-Key Encryption

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Outline

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Pseudo-Random Generators PRGs

- *Pseudorandomness* is a property of a distribution on strings. Say you have a distribution X on ℓ-bit strings that assigns some probability to every string in $\{0,1\}^{\ell}$. Pseudorandomness means that sampling form X is indistinguishable from sampling a uniform string of length ℓ.
- Ideally, we want a PRG to efficiently produce, from short seeds, long sequences of bits that appear to be generated by successive flips of a fair coin.
- Unpredictability is a very important property of sequences of coin tosses. Pseudo-random sequences should be unpredictable to computers with feasible resources. Given a sequence of $(n-1)$ bits, can you guess the n^{th} bit with a probability better than $1/2$?

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Pseudo-Random Generators PRGs

- A PRG is an efficient deterministic algorithm that expands a short, uniform seed into a longer, "uniform-looking" output.
- Informally, a PRG is cryptographically secure, if it passes all efficient statistical tests.

Definition

Let $G : \{0,1\}^n \to \{0,1\}^{\ell(n)}$ a deterministic polynomial-time algorithm where $\ell(n) > n$. G is a secure pseudorandom generator if $\forall$ probabilistic poly-time distinguisher (also called statistical test) D, the advantage

$$\mathrm{Adv}^{\mathrm{prg}}_{G,D}(n) = |\Pr[D(r) = 1] - \Pr[D(G(s)) = 1]| \leq \mathsf{negl}(n)$$

where the probabilities are taken over uniform choice of $s \in \{0,1\}^n$, $r \in \{0,1\}^{\ell(n)}$ and the randomness of D.

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Pseudo-Random Generators

- Do PRGs exist? Can we construct them?
- Not if $NP = P$
- What is the weakest assumption under which we can construct PRGs?
- It is the existence of *one-way functions* (i.e. easy to compute-hard to invert)

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PRG: examples

- *Linear Congruential Generator* (Non-Crypto PRG)
  - $X_{n+1} = aX_n + b \mod m$, where a, b, m are the constants and X_0 is the seed.
- Cryptographically suitable PRGs: /dev/random, Fortuna, Intel RdRand (available in Ivy Bridge processors), etc.
  - They continuously add entropy to internal state
  - Example of entropy source: Timing (hardware interrupts, e.g. keyboard, mouse, etc.).

Security of PRGs

- Attackers abilities: What they can observe or influence/manipulate in the inputs/outputs/state.
- Types of attacks:
  - Input Based Attacks (security goals: minimize the number of possible outputs, so guessing becomes easier, or force the generator to produce a particular output),
  - State Based attacks (security goals: backward/forward secrecy, i.e. predict past/future outputs).

- Remember, the goal is to “make our *secure* OTP practical”...
- OTP is perfectly secure. But $|k| \geq |m|$...!
- What if we use pseudo-random generators?
- And use a different security definition...
- (t, ϵ)-indistinguishability: We allow that the security may fail with probability $\leq \epsilon$ and we restrict attention to adversaries running in time $\leq t$.
- This is the basic idea of a new security definition called *semantic security* or *indistinguishable encryptions*!
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Fixed-length Encryption scheme using a PRG

Let G be a pseudorandom generator with expansion factor ℓ. For messages of length ℓ, we define the following encryption scheme $E = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$:

- $\mathsf{KeyGen}(n)$: It randomly picks random bit string of length n, i.e. $k \in \{0,1\}^n$.
- Enc : it takes as input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^{\ell(n)}$, it outputs

$$c \leftarrow \mathsf{Enc}(G(k), m) = G(k) \oplus m$$

- Dec : it takes as input a key $k \in \{0,1\}^n$ and a ciphertext $c \in \{0,1\}^{\ell(n)}$, it outputs

$$m \leftarrow \mathsf{Dec}(G(k), c) = G(k) \oplus c.$$

Semantic Security

Security Game

Challenger Ch | **Adversary** A

$\xleftarrow{m_0, m_1, |m_0| = |m_1|}$

$b \leftarrow\!\!\$ \{0, 1\}$

$\xrightarrow{c = \mathsf{Enc}(k, m_b)}$ Outputs his guess b'

Definition

An encryption scheme is semantically secure if for all efficient $\mathcal{A}$ the following holds:

$$\mathsf{Adv}^{\mathrm{eav}}_{E,\mathcal{A}}(n) = \Pr[\mathsf{A}\ \textit{wins}] - 1/2 \leq \mathsf{negl}(n)$$

What about security for multiple encryptions?

Semantic Security (also called Computational indistinguishability)

More formally,

- Let us distinguish between the two cases of $b = 0$ or $b = 1$. Call the first EXP(0) and the second EXP(1).
- For $b = 0, 1$, let G_b be the event that the output of the experiment EXP(b)=1. Now

$$\mathsf{Adv}^{\mathrm{SS}}_{E,\mathcal{A}}(n) = |\Pr(G_0) - \Pr(G_1)|$$

Theorem

If G is a secure PRG, then the encryption scheme derived from G is semantically secure.

Lemma

If $\mathcal{A}$ is an adversary against a semantic secure encryption scheme E , then there exists an adversary $\mathcal{B}$ against the PRG G of E s.t.

$$\mathsf{Adv}^{\mathrm{ss}}_{E,\mathcal{A}}(n) \leq 2 \cdot \mathsf{Adv}^{\mathrm{prg}}_{G,\mathcal{B}}(n)$$

Proof.

(By reduction)

Let G_b the event that the adversary outputs $b' = 1$ if the challenger uses the PRG while encrypting and R_b is the same but when using a truly random generator.

$$\begin{aligned}
\mathsf{Adv}^{\mathrm{ss}}_{E,\mathcal{A}}(n) &= |\Pr(G_0) - \Pr(G_1)| \\
&= |\Pr(G_0) - \Pr(R_0) + \Pr(R_0) - \Pr(R_1) \\
&\quad + \Pr(R_1) - \Pr(G_1)| \\
&\leq |\Pr(G_0) - \Pr(R_0)| + \underbrace{|\Pr(R_0) - \Pr(R_1)|}_{= 0 \text{ as OTP is sem. secure.}} \\
&\quad + |\Pr(R_1) - \Pr(G_1)| \\
&= 2\mathsf{Adv}^{\mathrm{prg}}_{G,\mathcal{B}}(n).
\end{aligned}$$

□

Stream Ciphers

- Terminology is not standard: it is either considered to be practical instantiations of pseudo-random generators or the encryption scheme which uses it.
- They produce as many random bits as exactly needed.
- They are more flexible (no upper bound on the number of bits) and efficient (each application takes the exact number of random bits that it requests)
- A stream cipher consists of two main deterministic algorithms:
- Init(s, IV): takes a seed s and an optional *initialization vector IV* and outputs an initial state st_0
- GetBits(st_i): takes the i-th state information st_i and outputs a bit y and an updated state, i.e. st_{i+1}

Stream Ciphers

Construction of a PRG G_ℓ:

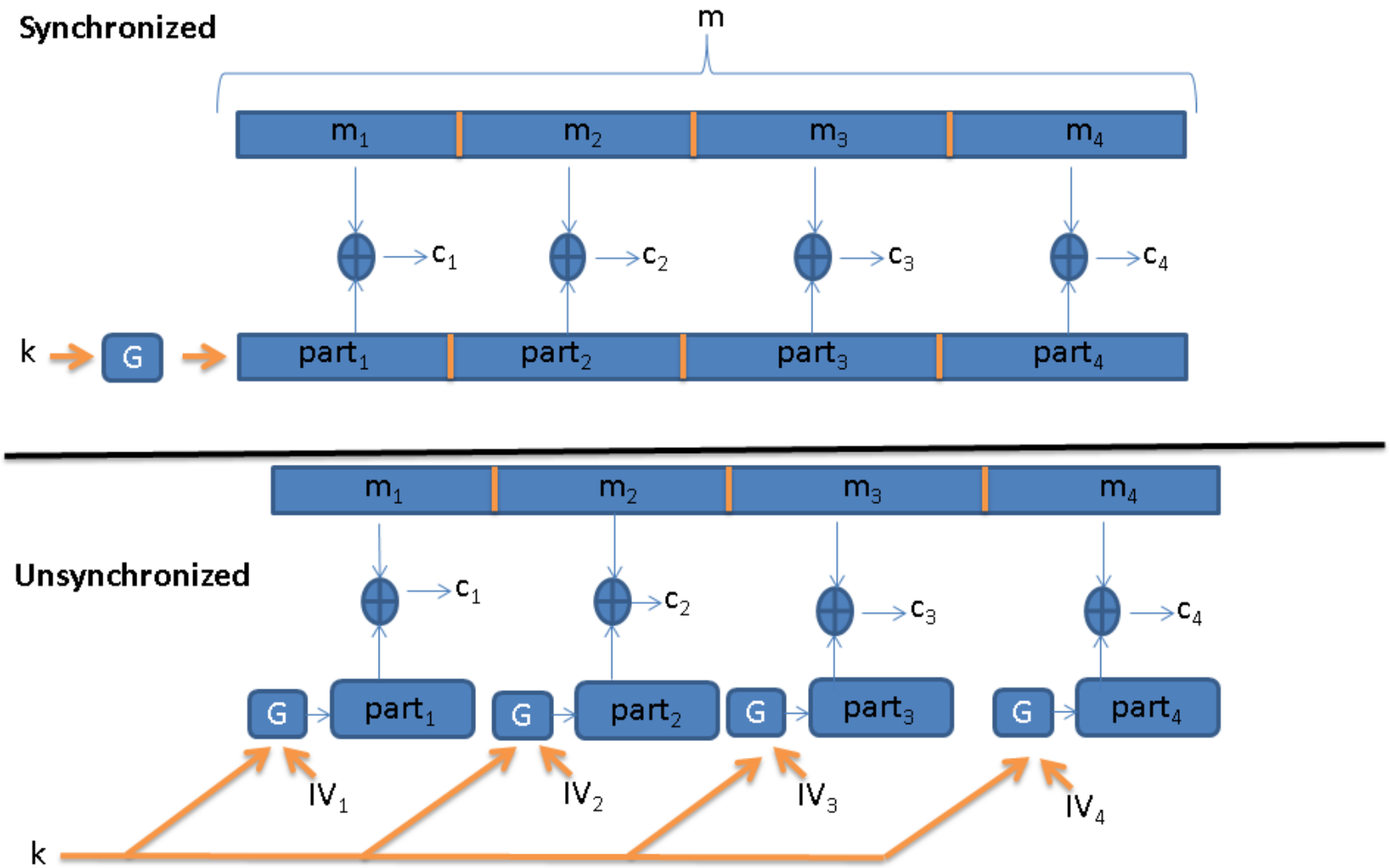
$\mathsf{st}_0 \leftarrow \mathsf{Init}(s, IV)$

for $i = 1, \cdots, \ell$;

$\quad (y_i, \mathsf{st}_i) \leftarrow \mathsf{GetBits}(\mathsf{st}_{i-1})$

return $y_1, \cdots, y_\ell$

Stream Cipher Mode of Operations



Stream Cipher Modes of Operations

- synchronized mode.
  - It gives a stateful CPA-secure encryption scheme.
  - Sender and Receiver must be synchronized.
  - It generates a long pseudo-random stream- different parts of it are used to encrypt different messages.
  - Therefore, messages should be received in order.
- unsynchronized mode.
  - It needs initialization vectors.
  - It gives *stateless* CPA-secure encryption.

Examples of Stream Ciphers

- Linear-Feedback Shift Registers (LFSR)
- RC4 by Ron Rivest 1987 (recent attack: AlFardan et al. 2013)
- eStream: Salsa 20, ChaCha (2008), and SOSEMANUK.
- eStream competition page:
  `http://competitions.cr.yp.to/estream.html`

RC4

- The state in RC4 consists of the triplet (S, i, j). S is a 256-byte array that contains a permutation of the numbers $0, \cdots, 255$. both $i, j \in \{0, \cdots, 255\}$.
- The key can be up to 256 byte long.

RC4- the Init() algorithm

Input: a 16-byte key

Output: Initial state (S, i, j)

for $i = 1, \cdots, 255$:

$S[i] \leftarrow i$ ▶ it sets S to the identity permutation

$k[i] \leftarrow k[i \mod 16]$ ▶ it expands the key to 256 bytes by repetition

$j = 0$

for $i = 1, \cdots, 255;$

$j \leftarrow j + S[i] + k[i] \mod 256$

Swap $S[i]$ and $S[j]$ ▶ "pseudo-random" swapping of S's elements

$i \leftarrow 0, j \leftarrow 0$

return (S, i, j)

RC4- the GetBits() algorithm

Input: Current state (S, i, j)

Output: byte y**, updated state** (S, i, j)

$i \leftarrow i + 1 \mod 256$

$j \leftarrow j + S[i] \mod 256$ ▶ changing j in a “pseudo-random” way

Swap $S[i]$ and $S[j]$

$t \leftarrow S[i] + S[j] \mod 256$

$y \leftarrow S[t]$

return $(S, i, j), y$

RC4: Security Analysis

- Weaknesses in its key scheduling algorithm, i.e. the Init() algorithm.
- Biases in the second output byte of RC4: the probability that it is 0 is $1/128$ instead of $1/256$ for $S = 256$.
- Biases in further bytes: *double-byte* or *adjacent-byte* biases.
- Conclusion: not secure, nevertheless, "its usage is still running at about 30% of all TLS traffic" (Garman et al. March 2015)

RC4: Security Analysis [AlFardan et al. 2013]

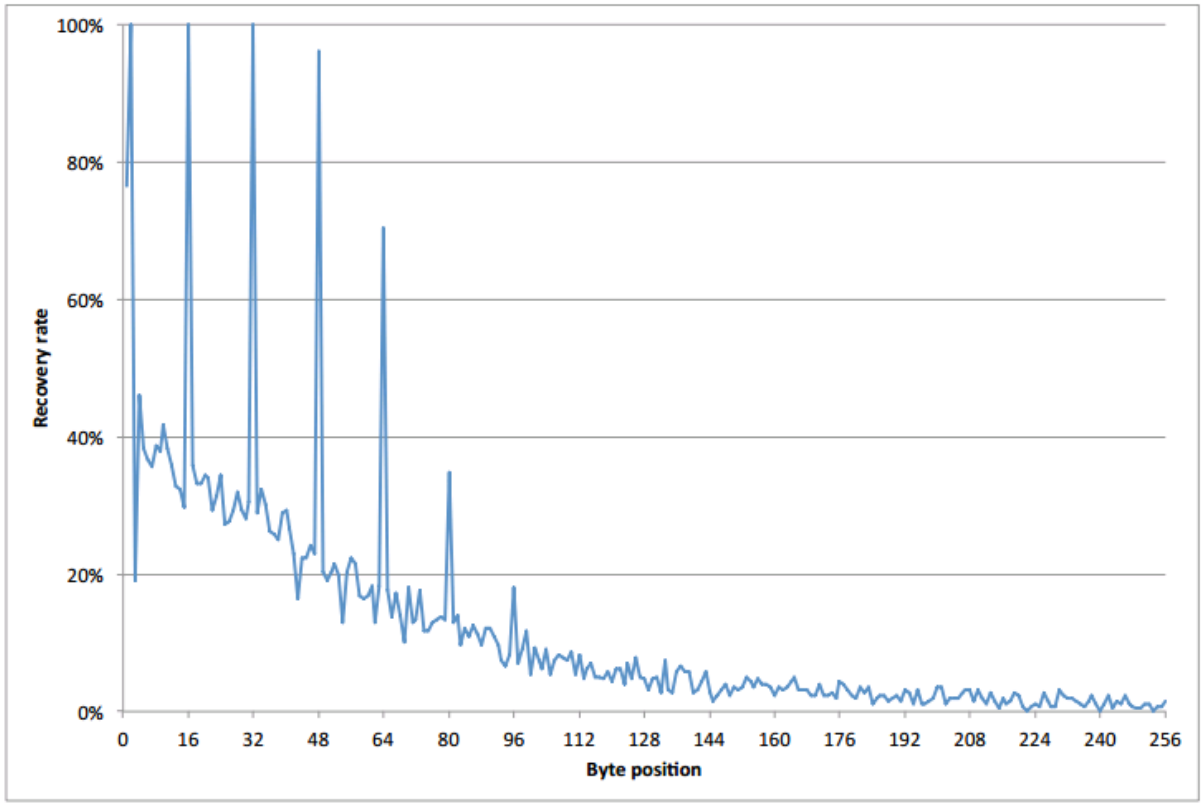


Figure: Recovery rate of the single-byte bias attack (based on 256 experiments)

More security definitions are needed...

- Stream ciphers are semantically secure.
- But what about multiple encryptions?
- What if the adversary wants to be challenged on two vectors of messages instead of two single messages?
- Obviously, he can trivially win the game (why?)
- Conclusion: deterministic encryption schemes are NOT secure under the multiple encryptions model.

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CPA Indistinguishability Experiment $\text{PrivK}^{\text{cpa}}_{\mathcal{A},E}$

Challenger Ch	**Adversary** A
$k = \text{KeyGen}(n)$	Access to an oracle $\text{Enc}(k, \cdot)$
$\xleftarrow{m_0, m_1, \lvert m_0\rvert = \lvert m_1\rvert}$	
$b \leftarrow \{0, 1\}$	
$\xrightarrow{c = \text{Enc}(k, m_b)}$	Outputs his guess b'

Definition

An encryption scheme is CPA-secure if for all efficient $\mathcal{A}$ the following holds:

$$\text{Adv}^{\text{cpa}}_{\mathcal{A},E}(n) = \Pr[\text{PrivK}^{\text{cpa}}_{\mathcal{A},E}(n) = 1] - 1/2 = \text{negl}(n)$$

Where $\text{PrivK}^{\text{cpa}}_{\mathcal{A},E}(n) = 1$ if $b' = b$, and 0 otherwise.

Note: CPA-secure $\Rightarrow$ CPA secure for multiple encryptions.

CCA Indistinguishability Experiment $\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},E}$

Challenger Ch

$k = \mathsf{KeyGen}(n)$

$\xleftarrow{m_0, m_1, |m_0| = |m_1|}$

$b \leftarrow \{0, 1\}$

$\xrightarrow{c = \mathsf{Enc}(k, m_b)}$

Adversary A

Access to two oracles $\mathsf{Enc}(k, \cdot), \mathsf{Dec}(k, \cdot)$

Access to two oracles $\mathsf{Enc}(k, \cdot), \mathsf{Dec}(k, \cdot)^c$

Outputs his guess b'

Definition

An encryption scheme is CCA-secure if for all efficient $\mathcal{A}$ the following holds:

$$\mathsf{Adv}^{\mathsf{cca}}_{\mathcal{A},E}(n) = \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},E}(n) = 1] - 1/2 = \mathsf{negl}(n)$$

Outline

Pseudo-Random Functions

- A generalization of the notion of pseudo-random generators.
- We now consider a “random-looking” function.
- It is the pseudo-randomness of a distribution on functions.
- We are interested in *keyed functions*, i.e. $F : \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$.
- Once we choose k, we fix it, and then use $F_k : \{0,1\}^* \to \{0,1\}^*$.
- F is *length-preserving* if the lengths of the key, input, output are equal.
- F is pseudo-random if the function F_k, for a uniform key k, is indistinguishable from a function chosen uniformally at random from the set of all functions with the same domain and range.

Pseudo-Random Functions

Definition

Let $Func[X, Y]$ be the set of all functions from X to Y. A function $F : K \times X \to Y$ is a secure Pseudo-Random Function (PRF) if F is efficiently computable and for all PPT distinguishers $\mathcal{A}$

$$\mathsf{Adv}^{\mathrm{prf}}_{F,\mathcal{A}}(n) = |\Pr[\mathcal{A}^{F()}(n) = 1] - \Pr[\mathcal{A}^{F_k()}(n) = 1]| < \mathsf{negl}(n)$$

where $F \in Func[X, Y]$, $k \in K$, and $\mathcal{A}$ has access to the function in question, i.e. either $F()$ and $F_k()$.

Note that $|Func[X, Y]| = |X|^{|Y|}$.

Pseudo-Random Permutations

A pseudo-random function $F : K \times X \to Y$ is an efficient pseudo-random permutation if the following hold:

- F is injective and $|X| = |Y|$.
- F is deterministic and efficiently computable.
- F^{-1} is efficiently computable.

In practice: $F : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$, where;

- 3DES: $n = 64$ bits, $k = 168$ bits
- AES: $n = 128$ bits, $k = 128, 192, 256$ bits

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Encryption using PRP

Let $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a pseudo-random permutation. We define the following encryption scheme $E = (\text{KeyGen}, \text{Enc}, \text{Dec})$:

- KeyGen : it takes n and outputs a key $k \in \{0,1\}^n$.
- Enc: it takes a key $k \in \{0,1\}^n$ and message m, it picks a random $r \leftarrow \{0,1\}^n$, and outputs

$$(c_0, c_1) \leftarrow (r, F_k(r) \oplus m).$$

- Dec: it takes a key k and a ciphertext $c = (c_0, c_1)$ and outputs

$$m \leftarrow (F_k(c_0) \oplus c_1).$$

Theorem

If F is a pseudo-random permutation then the encryption scheme E is CPA-secure.

Proof.

We will first prove that a version E' of this encryption scheme would be indeed CPA-secure if the function F was truly random, and then show that if E was insecure then we can distinguish F from a truly random function.
For E', there are two cases:

- r_c didn't appear in any of the encryption queries, and therefore the probability to win the game in this case is exactly 1/2.
- r_c appeared in *at least* one of the queries. Assuming that the adversary is restricted to $q(n)$ queries, then the probability of this event is at most $q(n)/2^n$.

Thus $\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},E'}(n) = 1] \leq 1/2 + q(n)/2^n$.

□

Proof.

F is a PRP $\Rightarrow$

$$\left| \Pr[\mathcal{A}^{F()}(n) = 1] - \Pr[\mathcal{A}^{F_k()}(n) = 1] \right| < \mathsf{negl}(n)$$

One can easily see that $\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},E'}(n) = 1] = \Pr[\mathcal{A}^{F()}(n) = 1]$ and $\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},E}(n) = 1] = \Pr[\mathcal{A}^{F_k()}(n) = 1]$. Thus

$$\left| \Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},E}(n) = 1] - \Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},E'}(n) = 1] \right| \leq \mathsf{negl}(n)$$

Therefore

$$\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},E}(n) = 1] \leq 1/2 + q(n)/2^n + \mathsf{negl}(n)\,.$$

□

Strong Pseudo-Random Permutations

Definition

Let Perm_n *be the set of all permutations from* $\{0,1\}^n$ *to* $\{0,1\}^n$*. Let* $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ *be an efficient length-preserving, keyed permutation.* f *is a strong pseudo-random permutation if for all probabilistic polynomial-time distinguishers D, there exists a negligible function negl such that*

$$|\Pr[D^{f(),f^{-1}()}(n) = 1] - \Pr[D^{f_k(),f_k^{-1}()}(n) = 1]| < \text{negl}(n)$$

where $f \in_R \text{Perm}_n$*,* $k \in_R \{0,1\}^k$*, and* D *has access to the functions in question, i.e. either* $f(), f^{-1}()$ *and* $f_k(), f_k^{-1}()$*.*

Notes:

- Strong PRP $\Rightarrow$ PRP
- $|\text{Perm}_n| = 2^n!$

Further Reading (1)


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