

## Prelims Introductory Calculus MT 2019: Sheet 2

1. Find solutions of the following ODEs with their associated initial conditions:

(a)

$$\frac{dy}{dx} + xy = x, \quad y(0) = 0;$$

(b)

$$2x^3 \frac{dy}{dx} - 3x^2 y = 1, \quad y(1) = 0;$$

(c)

$$\frac{dy}{dx} - y \tan x = 1, \quad y(0) = 1.$$

2. Solve the following differential equations:

$$(a) \quad (1 - x^2) \frac{dy}{dx} + 2xy = (1 - x^2)^{3/2}, \quad (b) \quad \frac{dy}{dx} - (\cot x)y + \csc x = 0.$$

3. By treating  $y$  as the independent variable, solve

$$(x + y^3) \frac{dy}{dx} = y, \quad y(1) = 2.$$

4. Find the general solutions of the following homogeneous linear equations:

$$(a) \quad \frac{d^2 y}{dx^2} - y = 0; \quad (b) \quad \frac{d^2 y}{dx^2} + 4y = 0; \quad (c) \quad \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 0;$$

$$(d) \quad \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 3y = 0; \quad (e) \quad \frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = 0; \quad (f) \quad \frac{d^4 y}{dx^4} + \frac{d^2 y}{dx^2} = 0.$$

5. By means of the substitution  $z = \ln x$  find the solution of the boundary value problem

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0 \quad y(1) = 0, \quad y(e) = 1.$$

6. Find the general solutions of the following inhomogeneous equations:

(a)

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 4e^{3x} + 2 \sin x;$$

(b)

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 3y = x^2 + 2.$$