It turned out that the projector in L1 was broken, which is why I couldn't show you the video of the normal modes of the mass-spring system on Friday. This example is discussed on pages 32–34 of the printed lecture notes. You can access the video by clicking on this link to my homepage:

`http://users.ox.ac.uk/~math0391/teaching.html`

For the system of masses and springs recall that the general solution is

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} A \cos\left(\sqrt{\frac{k}{m}}t + \phi\right) + \begin{pmatrix} 1 \\ -1 \end{pmatrix} B \cos\left(\sqrt{\frac{3k}{m}}t + \theta\right) . \tag{0.1}$$

Here $x(t)$ and $y(t)$ denote the horizontal displacements of the two masses from their equilibrium positions, while A, B, ϕ and θ are integration constants.

In the bottom half of the video the system is set up so that initially $B = 0$ (by setting $x(0) = y(0) \neq 0$). In this case the motion of the masses is described by the first normal mode solution

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} A \cos\left(\sqrt{\frac{k}{m}}t + \phi\right) .$$

In particular, since $x(t) = y(t)$ the masses oscillate together, with angular frequency $\omega_+ = \sqrt{k/m}$.

In the top half of the video instead the system is set up so that initially $A = 0$ (by setting $x(0) = -y(0) \neq 0$). In this case the motion of the masses is described by the second normal mode solution

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} B \cos\left(\sqrt{\frac{3k}{m}}t + \theta\right) .$$

In particular, since $x(t) = -y(t)$ the masses oscillate in opposite directions, with the higher angular frequency $\omega_- = \sqrt{3k/m}$.

If the masses are instead started in a general position, so that A and B are both non-zero, their subsequent motion is described by the sum of the normal mode solutions in (0.1), which will look much more complicated than the individual normal mode motions.

Please send comments and corrections to `sparks@maths.ox.ac.uk`.