

- Construct a central $100(1 - \alpha)\%$ confidence interval for the unknown parameter μ based on a random sample of size n from a normal distribution with mean μ and variance 1.

If $\alpha = 0.05$ and the length of the interval is to be less than 1, how large must n be? What if the length is to be less than 0.1?

[In questions 2 and 3, respectively, first assume that it is permissible to replace p, θ by \bar{x} to obtain an estimate of the variance of $\hat{p}, \hat{\theta}$. Can you find confidence intervals without doing this?]

- Suppose X_1, \dots, X_n is a random sample from a Bernoulli distribution with probability $P(X_i = 1) = p$.
 - What is the variance of X_i ? Show that the estimator $\hat{p} = \bar{X}$ has expectation p and find its variance.
 - Using the central limit theorem construct a random variable which has an approximate standard normal distribution and indicate how this can be used to find a $100(1 - \alpha)\%$ confidence interval for p .
 - Fifty female black ducks from locations in New Jersey were captured and radio-tagged prior to severe winter months. Of these 19 died during the winter. Find a 95% confidence interval for the proportion surviving. Claims made by environmentalists suggested a 50-50 chance of survival. Is this reasonable?
- Suppose X_1, \dots, X_n are independent Poisson random variables each with mean θ . Assuming n is large and using the central limit theorem, construct (i) a central confidence interval for θ , and (ii) an upper confidence limit for θ , each with an associated confidence of $1 - \alpha$.
- Let X_1, \dots, X_n be i.i.d. Uniform $[0, \theta]$. Find the maximum likelihood estimator $\hat{\theta}$ of θ . Show that the distribution of $\hat{\theta}/\theta$ does not depend on θ and show that the interval $(\hat{\theta}, \hat{\theta}/\alpha^{1/n})$ is a $1 - \alpha$ confidence interval for θ .
- (a) Let X and Y be independent normally distributed random variables with means a and b and variances v and w . State, without proof, the distribution of $\kappa X + \lambda Y$ for $\kappa, \lambda \in \mathbb{R}$.

Consider a random sample $(L_1, R_1), \dots, (L_n, R_n)$ of eye pressure measurements in the left and right eyes of n patients.

- Suppose that L_j and R_j are independent and normally distributed with unknown mean μ and known variance σ^2 , for $j = 1, \dots, n$. Obtain the likelihood function of the sample and derive the maximum likelihood estimator of μ . Construct a $100(1 - \alpha)\%$ confidence interval for μ .
- Suppose now that for each j , we do not assume that L_j and R_j are independent. Instead we assume that

$$L_j = M_j + D_j \quad \text{and} \quad R_j = M_j - D_j$$

for independent M_j and D_j , where $M_j \sim N(\mu, \sigma_1^2)$ and $D_j \sim N(0, \sigma_2^2)$, with $\sigma_1^2 \geq \sigma_2^2$ and $\sigma_1^2 + \sigma_2^2 = \sigma^2$. Here μ is unknown but σ_1^2 and σ_2^2 are known.

Find the distribution of L_j and the distribution of R_j , and find the maximum likelihood estimator for μ in terms of L_j and R_j .

Assuming that $\sigma_1^2 = \sigma_2^2 = \sigma^2/2$, find a $100(1 - \alpha)\%$ confidence interval for μ .

Assuming now that $\sigma_1^2 > \sigma_2^2$, state qualitatively, without calculations, how this change will affect (i) the maximum likelihood estimator and (ii) the confidence interval.