BO1 History of Mathematics
Lecture I
Introduction

MT 2019 Week 1

Contact details

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Course webpage:
https://courses.maths.ox.ac.uk/node/36602

Summary

- Arrangements: lectures, classes, the nature of the course
- Some advice on reading and taking notes
- The nature of history
- How can we organise/break down the history of mathematics?
- Rough overview of the course
- The ancient roots of mathematics

Administrative matters, MT

- Lectures: Mondays 10am–12noon, weeks 1–8, AWB C1 (lecture slides to be posted online after each lecture)
- Classes: to be held in weeks 3, 5, 6, 7; Wednesdays 9:30–11:00am, Queen's, Lecture Room C (FQ5/1)
- Work: a short essay (1,000 words) and preparation of a discussion topic for each class. Essays to be submitted on Mondays at the start of lectures
- Sheets: details of weekly reading, and of essay and discussion topics, can be found on course webpage
- Assessment: two-hour written paper in TT 2020

Administrative matters, HT

Topic: complex numbers

- A reading course with 'seminars' in weeks 1–8
- In-depth study of original works by John Wallis, Jean-Robert Argand, William Rowan Hamilton, and others, relating to the gradual acceptance of complex numbers
- Three essays of 2,000 words each, and preparation of a discussion topic for each class
- Assessment: 3,000-word essay, topic revealed in week 7, to be submitted by 12noon on Monday, week 10

Advice on taking notes and writing essays

- See the notes on the O1 course webpage
- Pay particular attention to the sections on
  - citing sources,
  - bibliographies, and
  - plagiarism
- The importance of clear and accurate citations will be stressed throughout the course — these serve the same purpose as proofs in mathematical arguments

Taking notes

From reading:

- *background* reading is for information, not examination: it is important, but don't spend too long on it
- read the material (at least) twice
  - on the first reading, try to get a general feel for the material, its meaning and significance
  - on the second, take notes (see the online guidance)

In lectures:

- Don't try to take down every detail
- Instead, read ahead, listen, think, ask questions

The lectures and the reading will cover some of the same ground but are designed to be complementary

Recommended reading: the main texts

Jacqueline Stedall, *Mathematics emerging: a sourcebook 1540–1900*, Oxford University Press, 2008

and *either*

Victor Katz, *The history of mathematics: brief version*, Pearson, 2003

or

Victor Katz, *A history of mathematics: an introduction*, 3rd edition, Addison-Wesley, 2009

(College libraries may have earlier editions of the latter, but these do not differ significantly in content as far as this course is concerned.)

Recommended reading: other useful books

Jacqueline Stedall, *The history of mathematics: a very short introduction*, Oxford University Press, 2012

Benjamin Wardhaugh, *How to read historical mathematics*, Princeton University Press, 2010

John Fauvel and Jeremy Gray, *The history of mathematics: a reader*, Macmillan/Open University, 1987

June Barrow-Green, Jeremy Gray and Robin Wilson, *The history of mathematics: a source-based approach*, vol. 1, MAA Press, 2019

Further books (usually on specific topics) will be cited throughout the course

Recommended reading: other useful resources

Some biographical resources:

Dictionary of Scientific Biography (*DSB*): available in RSL and several college libraries

Oxford Dictionary of National Biography (*ODNB*): available online through OxLiP

There are many other general histories of mathematics available — you are encouraged to read widely, but please read critically

What is history (of mathematics)?

History, particularly the history of mathematics, is often written in a very 'narrative' way, as a sequence of events with dates, detailing the achievements of major figures, telling a linear story of how we got to where we are today

Although this approach can be useful as a first approximation to the real story, there are several things wrong with it ...

... history is much more than a catalogue of events

What is history (of mathematics)?

When we study history, we may start by addressing the what, the when, and the who, but we are also interested in the how, and, perhaps most importantly, the why

The major figures in the history of mathematics are not the only people to have contributed to mathematics, so we might need to expand our notion of what counts as 'mathematics'

The story of mathematics is not linear: there are false starts and dead-ends, twists and turns, parallel developments; it is not a story of relentless progress: there are fallow periods and mistakes — but these too have shaped mathematics in their own ways

Augustus De Morgan on the history of mathematics (1865)

It is astonishing how strangely mathematicians talk of the Mathematics, because they do not know the history of their subject. By asserting what they conceive to be facts they distort its history ... There is in the idea of every one some particular sequence of propositions, which he has in his own mind, and he imagines that that sequence exists in history ...

Usually not the case!

Warning!

All this being said ...

Within the constraints imposed by this course (not least the need to fit it into 16 lectures), it will be all too easy to slip into a linear narrative of significant results

But this is not a disaster, provided we remain aware that it is happening

How do we organise the history of mathematics?

- periods (ancient, mediaeval, *n*th century, ...)
- places/cultures (Greece, Islam, Britain, ...)
- people (Archimedes, Newton, Euler, Galois, Hilbert, ...)
- topics (geometry, algebra, topology, probability, ...)
- sources (manuscripts, letters, books, journals, websites, ...)
- institutions (Royal Society, universities, LMS, ...)
- conferences (international congresses, local seminars, ...)

An outline of the course

Week 1:	Introductory material: mathematics up to 1600
Week 2:	Analytic geometry and the origins of calculus
Week 3:	Newton's *Principia*; the further development of calculus
Week 4:	Infinite series; the beginnings of rigour
Week 5:	Algebra: from classical to modern
Week 6:	Rigour in real analysis
Week 7:	Complex analysis; linear algebra
Week 8:	Geometry; number theory

Organisation by period

This course deals with (largely European) mathematics during the period 1600–1900

At different points of the course, we will consider particular places, people, and topics.

But if we were to divide up the course by century, we might see the following:

Organisation by period: 17th century

Topics:

- new notation
- analytic (co-ordinate) geometry
- calculus
- infinite series
- mathematics applied to the physical world

People: Descartes, Fermat, Wallis, Newton, Leibniz, Huygens, l'Hôpital, ...

Organisation by period: 18th century

Topics:

- many applications of (and some problems with) calculus
- applications (and problems) of infinite series
- developments in algebra and number theory
- mathematics applied to the physical world

People: Bernoullis, Euler, d'Alembert, de Moivre, Laplace, Lagrange, ...

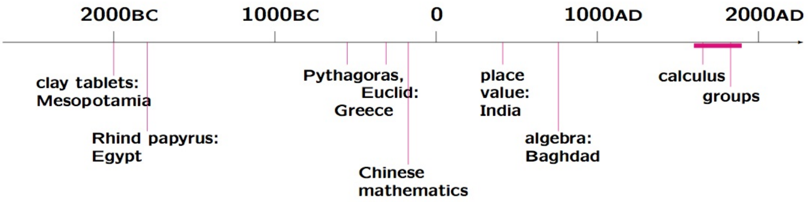
Organisation by period: 19th century

Topics:

- from calculus to analysis
- development of complex analysis
- rise of abstract algebra
- beginnings of linear algebra
- non-Euclidean geometry
- beginnings of axiomatisation

People: Gauss, Fourier, Bolzano, Cauchy, Abel, Galois, Dirichlet, Cayley, Dedekind, Cantor, ...

A timeline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Greece
Chinese
mathematics
place
value:
India
algebra:
Baghdad
calculus
groups

But what about non-Western mathematics?

Much of the world's present-day underlying mathematical culture — the way of doing it (e.g., notation, structure of arguments), of publishing it, and so on — is of European origin

The export of European mathematics to the rest of the world has been going on for centuries — for example, at the end of the 16th century, Jesuit missionaries began to introduce European mathematics into China, where it soon supplanted local mathematical traditions

In the period 1600–1900, most of the parts of the world that had a culture of mathematics that went beyond arithmetic were doing mathematics in a European style

Wasan

A major exception was wasan 和算 (Japanese mathematics) which flourished during the Edo Period (1603–1867)

See Tsukane OGAWA, 'A review of the history of Japanese mathematics', *Revue d'histoire des mathématiques* **7** (2001) 137–155

and also Japanese Mathematics in the Edo Period

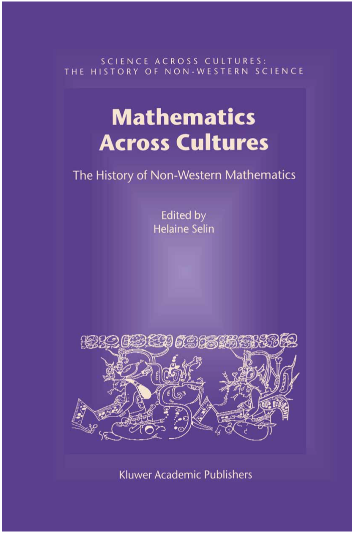
Other non-European mathematics

In this course, we will see other instances of non-European mathematics that are important for our story; for example:

- solution of systems of linear equations in the *Jiŭzhāng Suànshù* (2nd century BC, China) [week 7]
- solution of polynomial equations by al-Khwārizmī (9th century Baghdad) [week 5]
- study of infinite series by the 'Kerala School' (14th–16th-century India) [week 4]

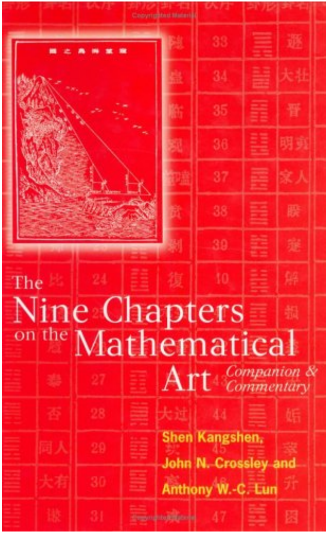
But we will not be systematic in our treatment of non-European mathematics

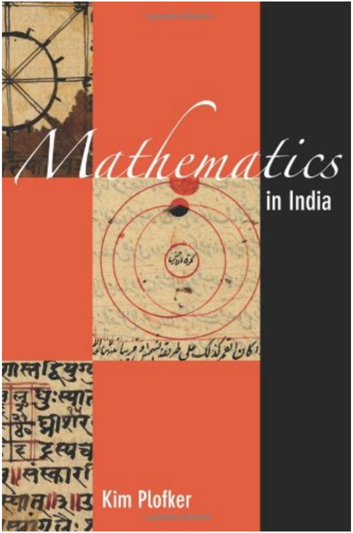
On non-European mathematics



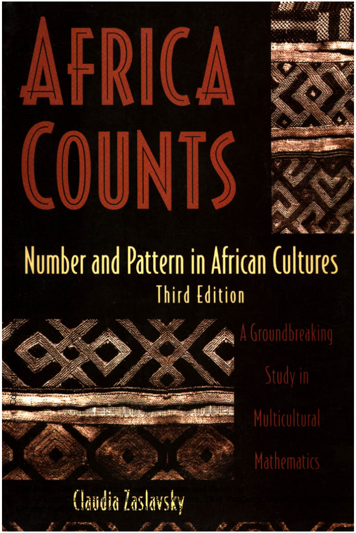
Helaine Selin (ed.), *Mathematics across Cultures: The history of non-Western mathematics*, Kluwer Academic Publishers, 2000

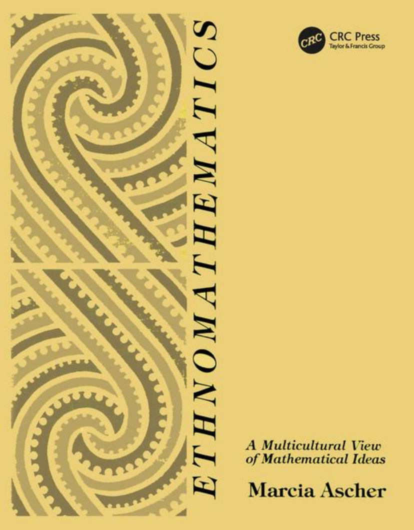
On non-European mathematics



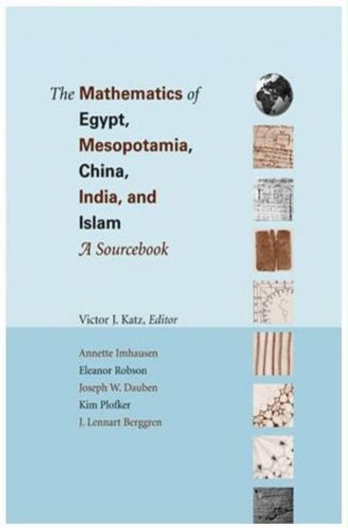


On non-European mathematics





Ancient origins of mathematics



On the ancient oriental origins of mathematics see:

Victor J. Katz (ed.), *The mathematics of Egypt, Mesopotamia, China, India, and Islam: a sourcebook*, Princeton University Press, 2007

Ancient Greek mathematics

Earliest origins of Greek mathematics in 6th century BC

But what do we mean by ‘Greek’?

500 BC – 300 BC	Collection of city-states in Greece
300 BC – AD 500	Greek-speaking peoples around the Mediterranean, especially in Alexandria

Some major figures of ‘Greek’ mathematics

Pythagoras	Samos (Greece)?	c. 600 BC
Euclid	Alexandria (Egypt)?	c. 300 (or 250?) BC
Archimedes	Syracuse (Sicily)	c. 250 BC
Apollonius	Perga (Turkey)	c. 180 BC
Diophantus	Alexandria (Egypt)	c. AD 200

Euclid's *Elements*

The 'elements of geometry' in 13 books, compiled around 300 (250?) BC from existing geometrical knowledge

Books I–VI	plane geometry	points, lines, angles, circles, ...
Books VII–X	properties of numbers	odd, even, square, triangular, prime, perfect, ...
Books XI–XIII	solid geometry	cube, tetrahedron, icosahedron, ...

Euclid's *Elements*, book I

23 definitions: point, line, surface, angle, circle, ...

5 postulates: what one can do
e.g. a straight line may be drawn between any two points;
a circle may be drawn with given centre and radius

5 'common notions': how one may reason
e.g. if equals are added to equals, then the wholes are equal

48 propositions: each built only on what has gone before

The influence of Euclid's *Elements*

HUGE influence on Western mathematics:

- hundreds of editions and translations from renaissance onwards
- basis of mathematics teaching in schools until c. 1960
- style: definitions—axioms—theorems—proofs
- status of 'Parallel Postulate' led to much investigation and, ultimately, non-Euclidean geometries
- problems of 'ruler and compass' construction inspired much investigation and many new discoveries
- wider cultural importance: http://readingeuclid.org/

Other Greek authors

Archimedes	d. 212 BC:	method of exhaustion and much else
Apollonius	c. 180 BC:	conic sections
Diophantus	c. AD 250:	*Arithmetica* in 13 books (number problems)

Also had HUGE influence on Western mathematics