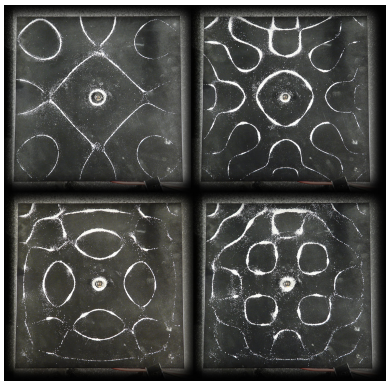
Random Plane Waves

Introduction

There are many problems in physics that are related to the study of eigenfunctions of Laplace operator and their zero-level sets. For example sand on a vibrating membrane concentrates on the zero-set of a Laplace eigenfunction. It is conjectured that for a very large class of domains, the typical behaviour of a high energy eigenfunction is the same as that of a random superposition of simple plane waves (RPW). This motivates our interest in RPW and their nodal lines (curves where the plane wave is equal to zero).

(a) Random spherical harmonic: an analogue of RPW on a sphere (figure by A. Barnett).



(b) Sand on a vibrating plate shows the zero-level lines of eigenfunctions of bi-Laplacian (photos from MIT Physics TSG site).

Project

There are several conjectures about RPW that can be numerically tested. They all require code that can effectively compute RPW and their nodal domains. The first part of the project will require sampling random plane wave (a code for this is available) and extracting geometric information from it. We will be mostly interested in nodal domain and nodal lines, but there other interesting features like critical points and gradient flow lines connecting them. The second part will involve numerical verification of several conjectures about these geometrical features. In particular we are interested in the nodal domains: what is the expected number of nodal domains inside given area, what is the variance of their number? Other interesting questions are: how many critical points there are? What is their distribution?

Prerequisites

You will require only basic knowledge of probability and analysis.

Reading

A nice survey can be found in the second part of

Nazarov and Sodin, *Random complex zeroes and random nodal lines*, Proceedings of the International Congress of Mathematicians. Volume III, 1450–1484, Hindustan Book Agency, New Delhi, 2010. Also available at `http://arxiv.org/abs/1003.4237`