Wednesday, 20 May 2020 18:49

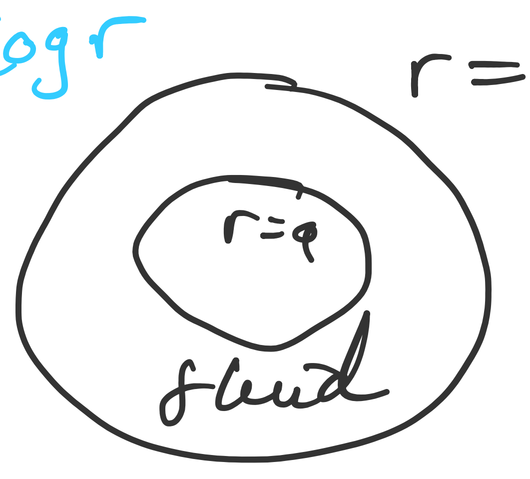
2014 Q(1)b $\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = \frac{G}{\mu}$, constant,

with $w=0$ on $r=b$ (outer wall)
$w=-U$ on $r=a$ (inner wall).

$$w = \frac{G}{4\mu}(r^2-b^2) + \frac{\log(r/b)}{\log(b/a)}\left(U - \frac{G}{4\mu}(b^2-a^2)\right)$$

$= A\log r$

iii) The drag force is
$\iint_{\text{body}} \underline{\underline{\sigma}}.\underline{n}\, dS$
in the z direction



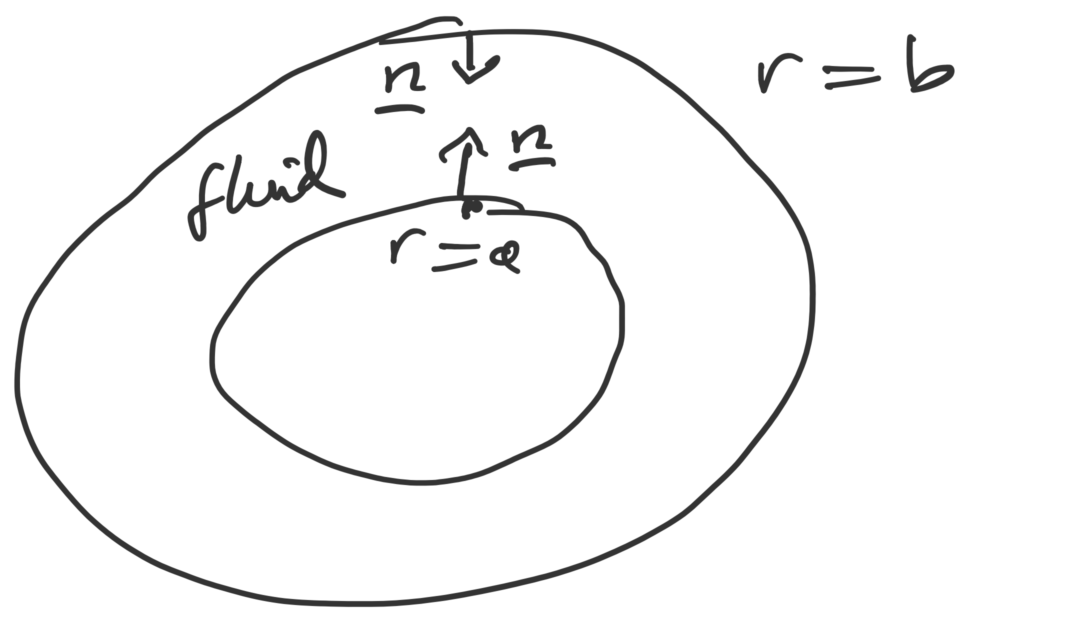
$\sigma_{13} = \mu\frac{dw}{dr}$
$= \mu\left(\frac{Gr}{2\mu} + \frac{A}{r}\right)$

The drag on the $r=a$ wall is
$\iint_{r=a} \sigma_{13}\, dS = 2\pi a\mu\left(\frac{Ga}{2\mu} + \frac{A}{a}\right)$
$= \pi G a^2 + 2\pi A\mu$

The drag on the $r=b$ wall is
$-\iint_{r=b} \sigma_{13}\, dS = -\pi G b^2 - 2\pi A\mu$

The total drag is $\pi G(a^2-b^2)$,
which is $G \times$ cross-sectional area.
This comes from $\nabla^2 w = \frac{G}{\mu}$ and the divergence theorem.
$$\iint_{\text{annulus}} \nabla\cdot(\nabla w)\, dS = \iint_{\text{annulus}} \frac{G}{\mu}\, dS$$
$$\int_{\partial(\text{annulus})} \mu\frac{\partial w}{\partial n}\, ds = G\iint_{\text{annulus}} dS$$
$$= G \times \text{cross-sectional area}.$$

$\underline{\underline{\sigma}}.\underline{n}$ is the force exerted by the fluid towards which $\underline{n}$ points.



$\underline{n} = \underline{e_r}$ on $r=a$
$\underline{n} = -\underline{e_r}$ on $r=b$

The only velocity component is w, and the only coordinate is r, so the only nonzero component of the viscous stress is $\sigma_{13} = \mu\frac{dw}{dr}$.

If w depended on θ, we'd need
$\int_{\partial V} \mu\frac{\partial w}{\partial n}\, ds$ where $\frac{\partial w}{\partial n}$ might have a $\frac{\partial w}{\partial \theta}$ term.

general
$= \int_{\partial V} \underline{k}.\underline{\underline{\sigma}}.\underline{n}\, ds$ using
$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \mu\left((\nabla\underline{u}) + (\nabla\underline{u})^T\right)$

In complete generality, the drag force is
$\iint_{\text{body}} \underline{\underline{\sigma}}.\underline{n}\, dS$
with $\underline{n}$ pointing into the fluid.

2014 Q3.
We're told that $\frac{\delta L W \rho}{\mu} << 1$,
as well as $\delta << 1$.

M determines h, and hence $\delta = \frac{h}{L}$.
We need M large enough to ensure $\delta << 1$.
However, W, L, ρ, μ are all prescribed and don't depend on M.
$\therefore$ δ sufficiently small $\Rightarrow \frac{\delta L W\rho}{\mu} << 1$.

We need $\delta << \min\left(1, \frac{\mu}{LW\rho}\right)$.

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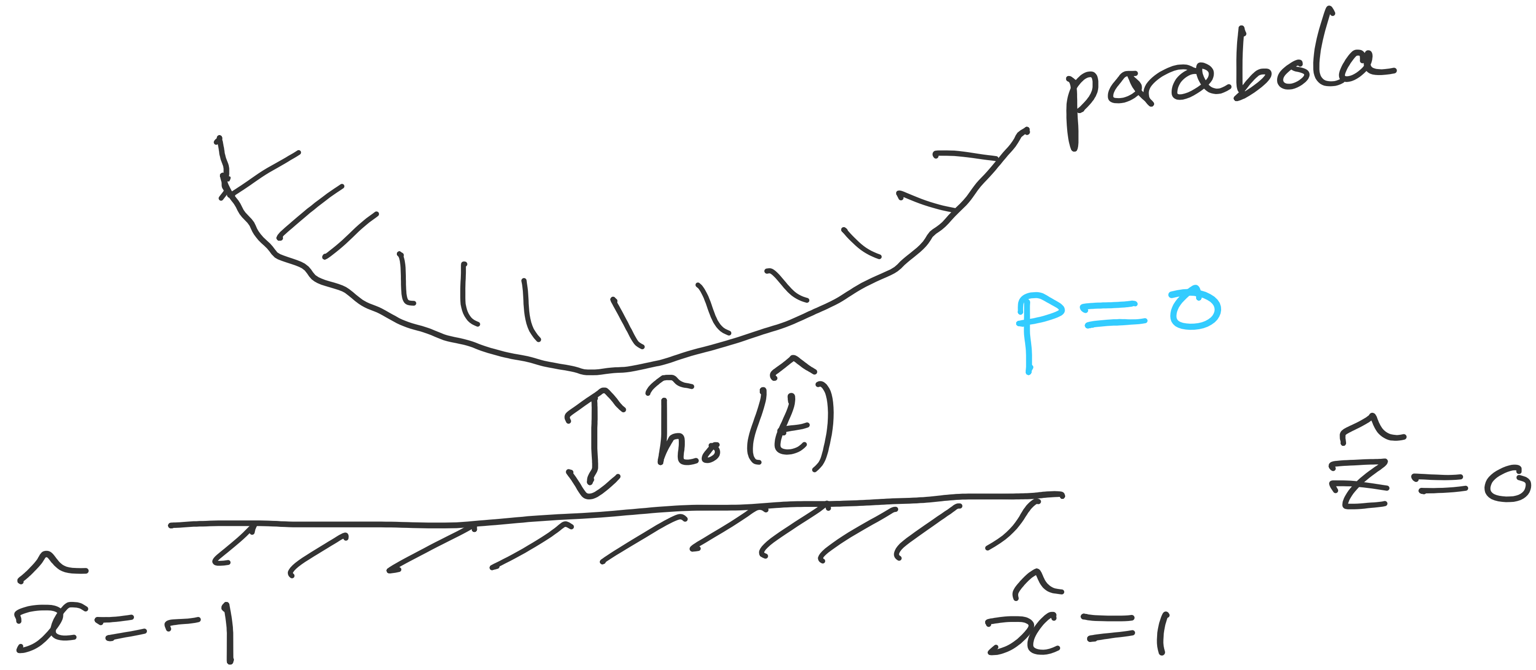
2013 Q3



Atmospheric pressure at $\hat{x}=1$, and also at $\hat{x}=-1$, so $\hat{p}(\pm 1,\hat{t})=0$.

This problem is symmetric about $\hat{x}=0$.
The pressure $\hat{p}$ is an even fn of $\hat{x}$.
$\Rightarrow$ $\frac{\partial \hat{p}}{\partial \hat{x}}$ is an odd function of $\hat{x}$
$\Rightarrow$ $\frac{\partial \hat{p}}{\partial \hat{x}}=0$ on $\hat{x}=0$ for all $\hat{t}$.

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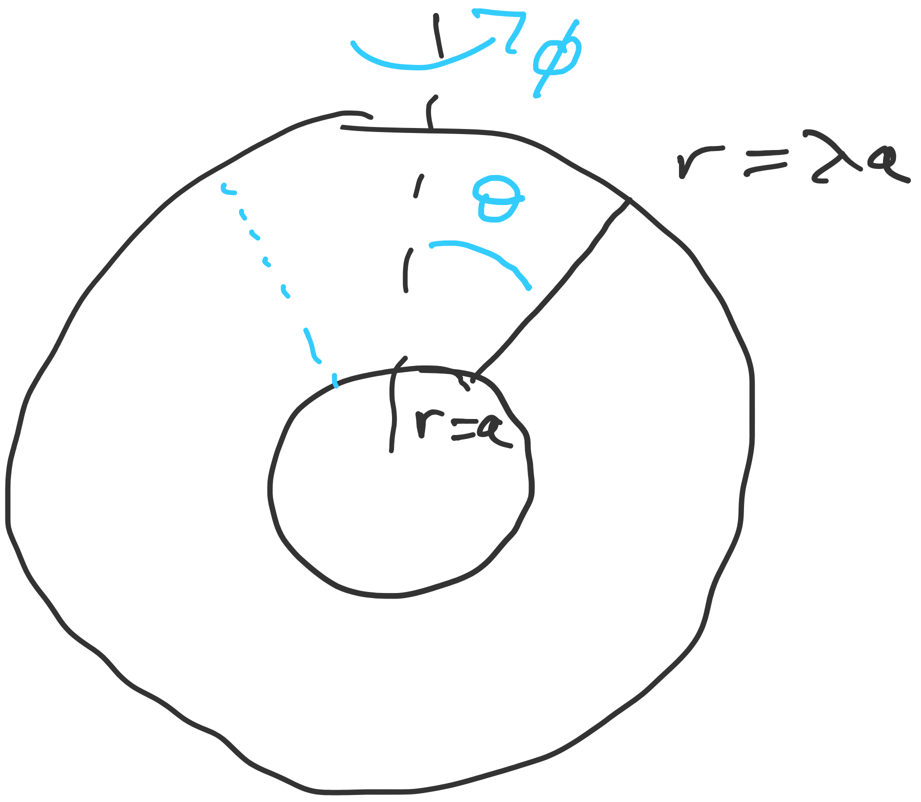
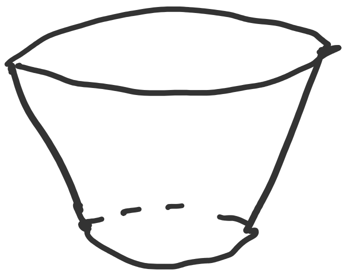
2017 Q3



The flux through a surface S is

$$\iint_S \underline{u}\cdot\underline{n}\,dS.$$

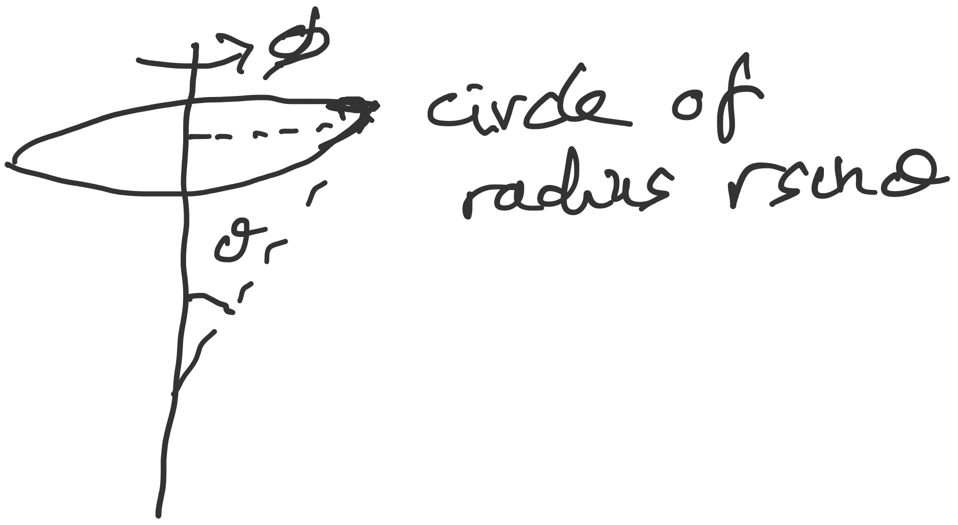
The surface $\theta=\theta_0$ is a truncated cone



The flux $F=\int_0^{2\pi}d\phi\int_a^{\lambda a}dr\,(r\sin\theta)\,u_\theta$

$$d\underline{r} = dr\,\underline{e_r} + r\,d\theta\,\underline{e_\theta} + r\sin\theta\,d\phi\,\underline{e_\phi}$$

In plane polars the area element would be $r\,dr\,d\phi$, for circles of radius r.



In spherical polars, changing ϕ moves around a circle of radius $r\sin\theta$, so our area element is $r\sin\theta\,dr\,d\phi$.

$$F=\int_0^{2\pi}d\phi\int_a^{\lambda a}dr\,(r\sin\theta)\left(-\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial r}\right)$$

$$=-2\pi\int_a^{\lambda a}\frac{\partial\Phi}{\partial r}\,dr$$

$$=-2\pi\left[\Phi\right]_{r=a}^{r=\lambda a}.$$