B1.1 Logic

MT 19

Problem Sheet ♯ 1

Please, if your class is in week 2, read the slides of Lectures 1 and 2 before attempting this sheet.

(1) (a) Which of the following are formulas of $\mathcal{L}$? Give reasons.

(i) $(p_3 \to p_1)$
(ii) $p_1 \to p_2 \to p_3$
(iii) $(\neg p_5 \wedge \neg p_6) = \neg p_{11}$
(iv) $(p \leftrightarrow \neg p)$
(v) $((p_1 \vee \neg p_1) \to (\neg p_2))$

(b) Prove carefully that for any formula ϕ, the number of left parentheses occuring in ϕ is equal to the number of right parentheses occuring in ϕ.

(2) (a) Prove that the length of a formula with exactly n occurences of the negation symbol and m occurrences of binary connectives is $4m + n + 1$. Check this for the formulas in question (1) (a).

(b) List all formulas of $\mathcal{L}$ of length ≤ 6.

(3) Can a proper initial segment of a formula ever be a formula again? How about final proper segments?

(4) Prove the Unique Readability Theorem.