Electromagnetism, Sheet 2: Boundary problems in Electromagnetism

1.- Two infinite, grounded, conducting planes are located at $x = a/2$ and $x = -a/2$. A point charge q is placed between the planes at the point (x', y', z'), where $-a/2 < x' < a/2$.

Find the location an magnitude of all the image charges needed to satisfy the boundary conditions on the potential and write down the Dirichlet Green's function $G_D(\vec{r}, \vec{r}')$.

2.- A conducting grounded sphere of radius a is placed on a uniform electric field E_0 along the z direction. Use the method of images to find the potential outside the sphere. Find the charge density induced on the surface of the sphere.

[A uniform electric field can be simulated by two opposite charges separated a distance d. In the limit in which d is very large, with an appropriate scaling of the charges q, we obtain a uniform electric field]

3.- Consider an insulated sphere of radius a where the potential in the upper hemisphere is kept at $+V$ while the potential at the lower hemisphere is kept at $-V$. Use the Dirichlet Green's function for the sphere found in the lectures to compute the potential for all the points in the $z-$axis, with $z > a$.

4.- A hollow cube has conducting walls defined by six planes $x = y = z = 0$ and $x = y = z = a$. The walls at $z = 0$ and $z = a$ are held at constant potential V. The other four sides are at zero potential. Find the potential $\Phi(x, y, z)$ at any point inside the cube.

5.- Use orthonormal functions to find the potential at all points outside a sphere of radius a, if you are told that the potential at the sphere is fixed to:

$$\Phi(a, \theta, \phi) = \frac{1}{\sqrt{4\pi}} \sin^2 \theta$$

6.- The Dirichlet Green's function in the region between two spherical shells of radius a and b is given by

$$G(\vec{r}, \vec{r}') = 4\pi \sum_{\ell,m} \frac{1}{(2\ell+1)\left(1 - \frac{a^{2\ell+1}}{b^{2\ell+1}}\right)} \left(r_<^{\ell} - \frac{a^{2\ell+1}}{r_<^{\ell+1}} \right) \left(r_>^{-(\ell+1)} - \frac{r_>^{\ell}}{b^{2\ell+1}} \right) Y^*_{\ell,m}(\theta', \phi') Y_{\ell,m}(\theta, \phi)$$

Use this to compute the potential in the region of interest, if the potential at the shells is kept constant $\Phi(a, \phi, \theta) = 0$ and $\Phi(b, \phi, \theta) = 1$. Could you have solved this problem in a simpler way?