

Numerical Solution of Differential Equations II. QS 6 (HT 2020)

1. Derive the first order upwind scheme for the initial value problem

$$\frac{\partial \mathbf{u}}{\partial t} + \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix} \frac{\partial \mathbf{u}}{\partial x} = 0, \quad \mathbf{u}(x, 0) = \mathbf{u}_0(x).$$

What is the CFL stability criterion in this case?

2. For the nonlinear conservation law

$$u_t + f(u)_x = 0$$

find a numerical flux for the Lax-Wendroff finite difference scheme so that it can be written in conservation form. Show also that the Lax Wendroff scheme for the first order wave equation can be written as

$$U_j^{n+1} = U_j^n - \nu(U_j^n - U_{j-1}^n) - \frac{1}{2}\nu(1 - \nu)(U_{j+1}^n - 2U_j^n + U_{j-1}^n).$$

3. Verify that the Lax-Freidrich's scheme is a TVD scheme for the first order wave equation when the CFL criteria are satisfied, but that the Lax Wendroff scheme does not satisfy the sufficient conditions for a TVD scheme except when $\nu = 0, \pm 1$.